Gears 2

Gear trains and applications

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Gears related Lecture sessions

Gears 1

- **Introduction to gears**
  - Functions & types
  - Gear terminologies & conjugate action
  - Involute profile, fundamental equations, tooth system

Gears 2

- **Gear trains (systems) and their applications**
  - **Simple and compound trains**
  - **Planetary train**
  - **Differential unit**
  - **Applications**

Gears 3

- **Gear stress analysis & design**
  - Common forms of gear failure
  - Gear force analysis
  - AGMA gear design analysis

Summary of Gears 2

- To be able to determine gear ratio of different gear trains
- To be able to effectively use simple sketches in design of gear trains
- To understand a few specific gear systems used in actual applications

Part 1:
- Simple and compound trains
- Worked examples

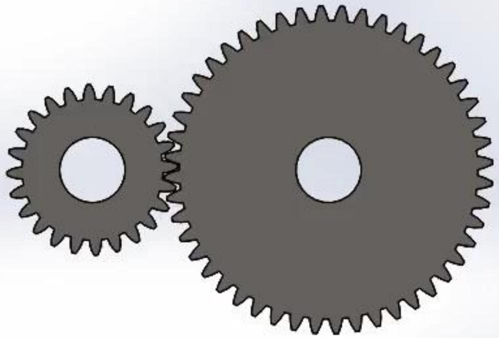
Part 2:
- Planetary trains
- Worked examples

Part 3:
- Differential unit
- Its application in cars

Recap of Gears 1: SW models of typical gear types

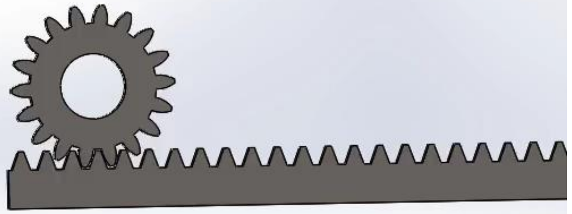
Gears 1 Part 1

Spur gears
Np=24, Ng=48

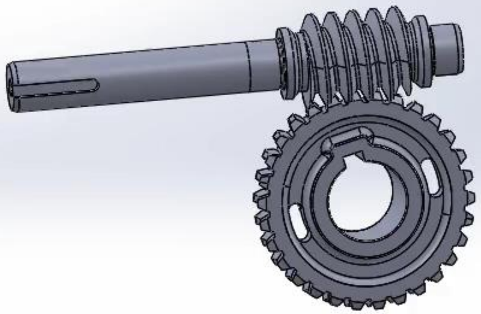


Internal spur gears
Np=18, Ng=72

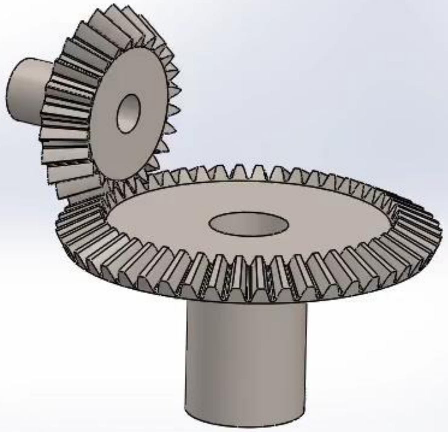
Rack & pinion
Np=20



Worm & gear
Ng=30



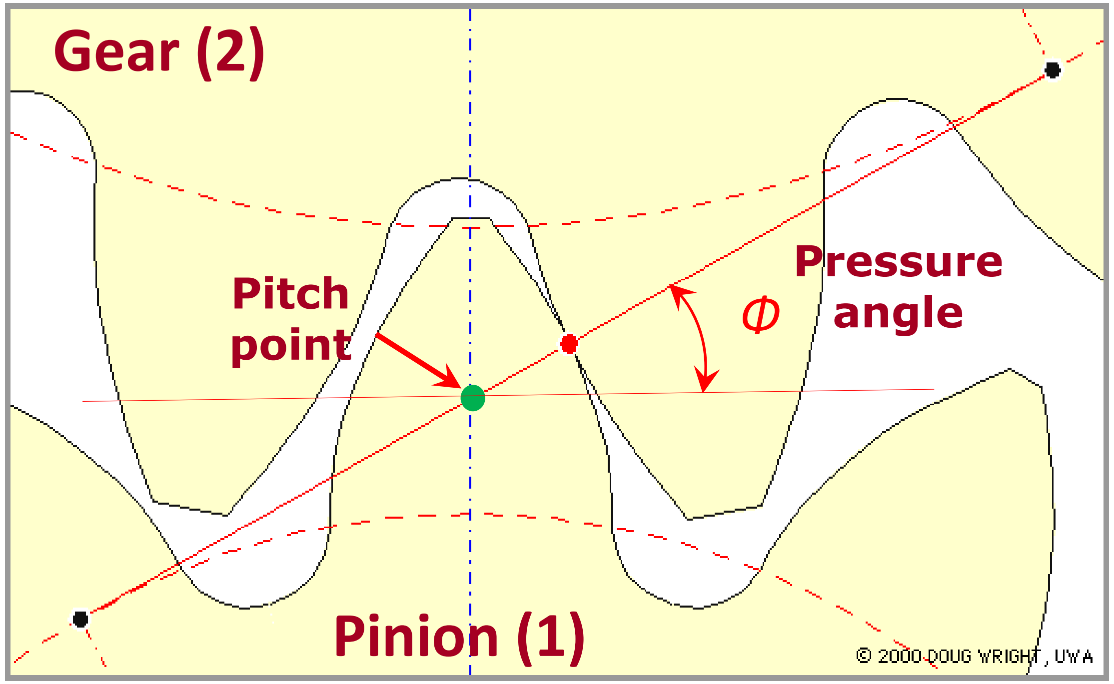
Bevel gears
Np=24, Ng=48



Note: SW assembly models of different gear types are available on Moodle

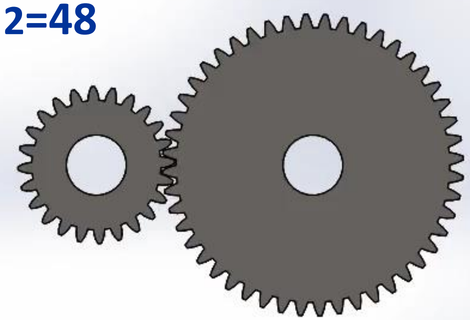
Recap of Gears 1

Involute gears engage in **conjugate action** to produce constant velocity ratio



http://www.mech.uwa.edu.au/DANotes/gears/meshing/meshing.html

Spur gears, m=1 mm
N1=24, N2=48



Internal spur gears, m=1 mm
N1=18, N2=72

Gear ratio equation: $$Z = \frac{\omega_1}{\omega_2} = \frac{d_2}{d_1} = \frac{N_2}{N_1}$$

Necessary & sufficient conditions :

$$m_1 = m_2, \quad \phi_1 = \phi_2$$

Note: The most commonly used pressure angle is **ϕ** or **α** = 20^0

Pitch diameter: d=mN,

m is Module, *in mm (SI unit)*

Simple train

Definition: A power transmission system consisting of gears is called a **gear train**.

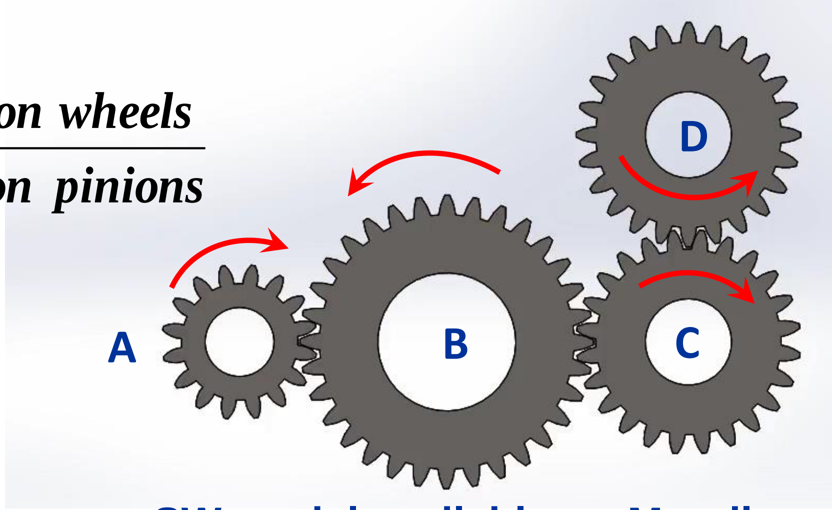
- In a simple train each shaft carries only one gear

- Velocity ratio:

$$Z = \frac{\omega_{In}}{\omega_{Out}} = \pm \frac{product\ of\ \ number\ of\ \ teeth\ on\ wheels}{product\ of\ \ number\ of\ \ teeth\ on\ \ pinions}$$

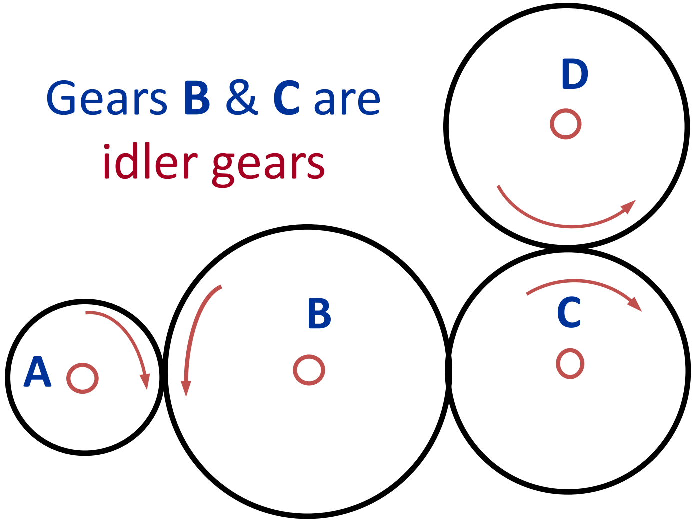
$$Z = \frac{\omega_A}{\omega_D} = -\frac{N_B N_C N_D}{N_A N_B N_C} = -\frac{N_D}{N_A}$$

NA=16, NB=32, NC=24, ND=24



SW model available on Moodle

Gears **B** & **C** are idler gears



A simple train

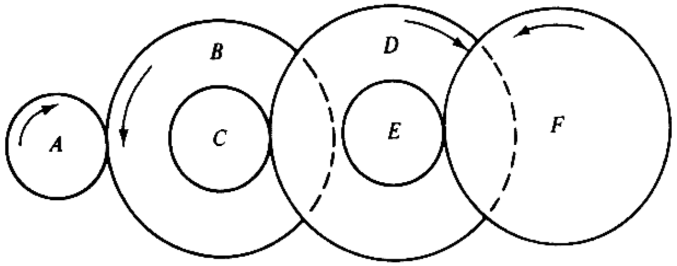
- **(+, -) signs** represent the same or opposite direction of the output gear with respect to the input gear.
- The intermediate or idler gears do not change the overall velocity ratio but can be used to fill up a gap of distance and to reverse the direction of the output shaft.

Compound train

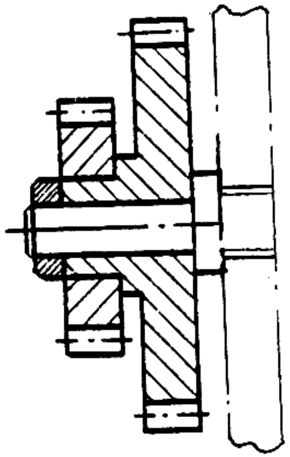
- In a **compound train** at least one or more shafts carry two gears secured concentrically.

- Velocity ratio:

$$Z = \frac{\omega_A}{\omega_F} = -\frac{N_B \cdot N_D \cdot N_F}{N_A \cdot N_C \cdot N_E}$$

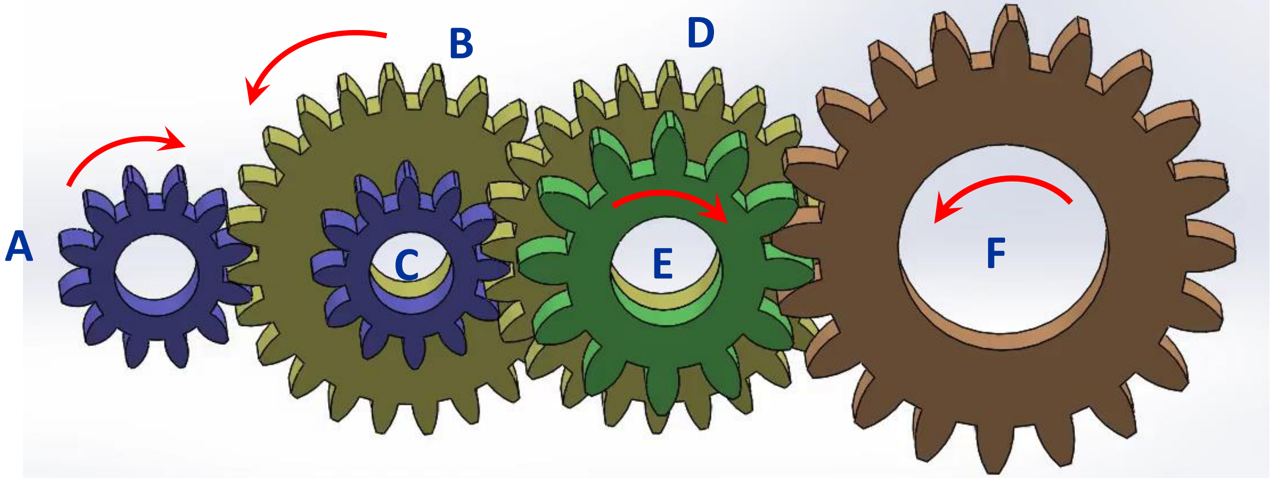


A compound train


Mounting of compound gears

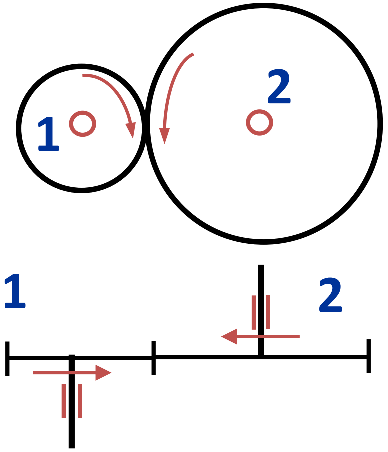
• Gears of a **compound train** are independent but locked together. They do not need to have the same module nor the same number of teeth.

N_A=12, N_B=24, N_C=12, N_D=24 (m=1mm), N_E=12 & N_F=20 (m=1.5mm)

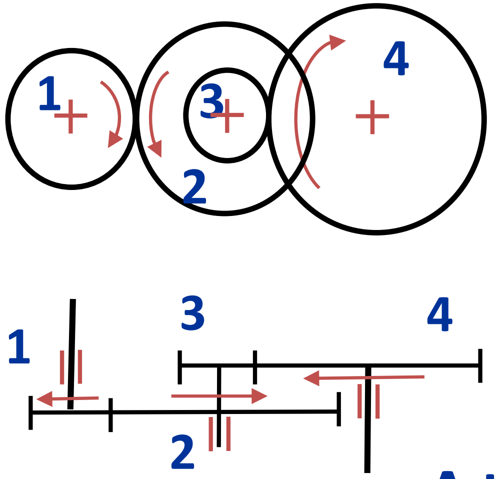


SW model available on Moodle

Gear train applications

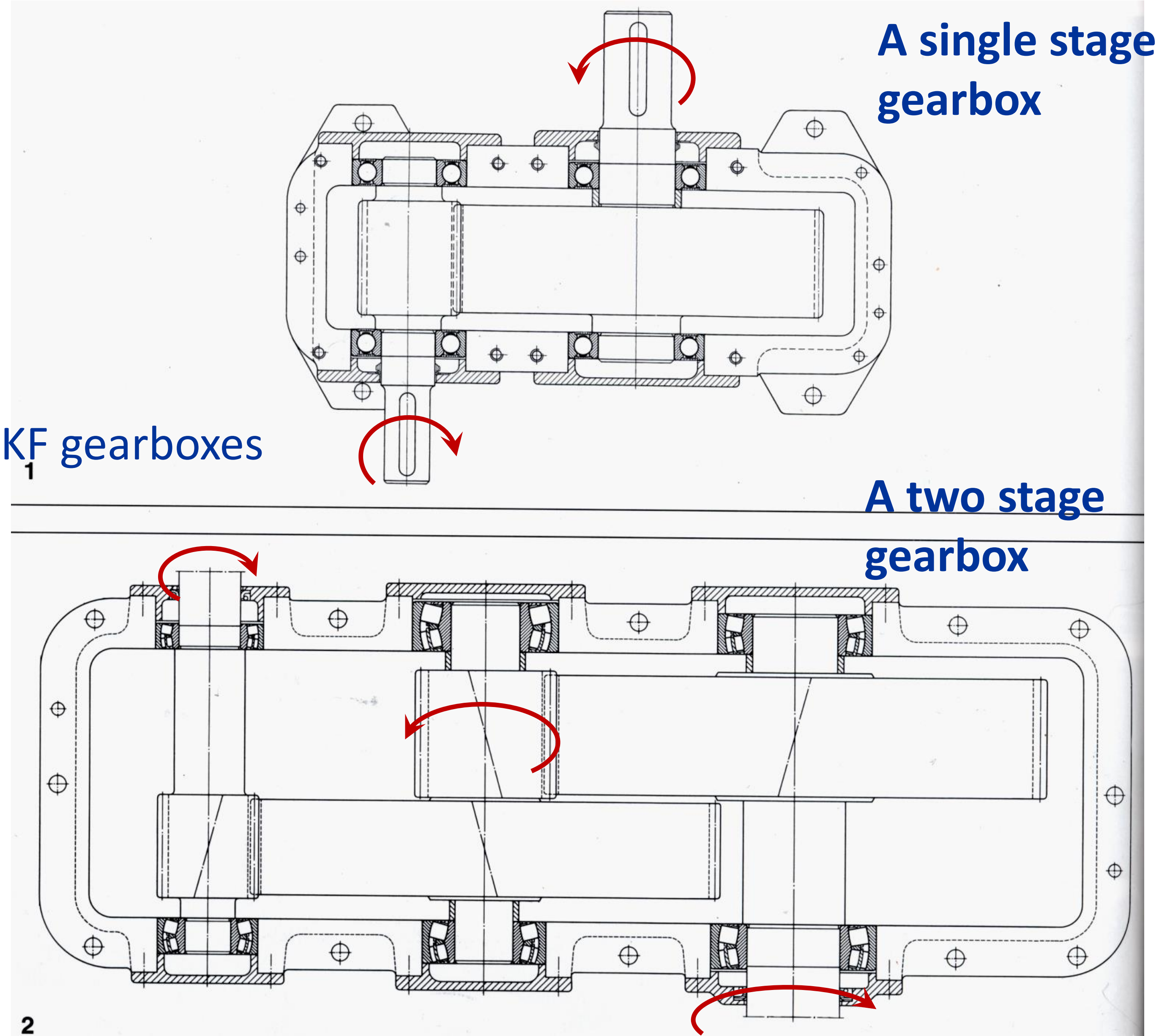


A simple train



A two stage compound train

SKF gearboxes

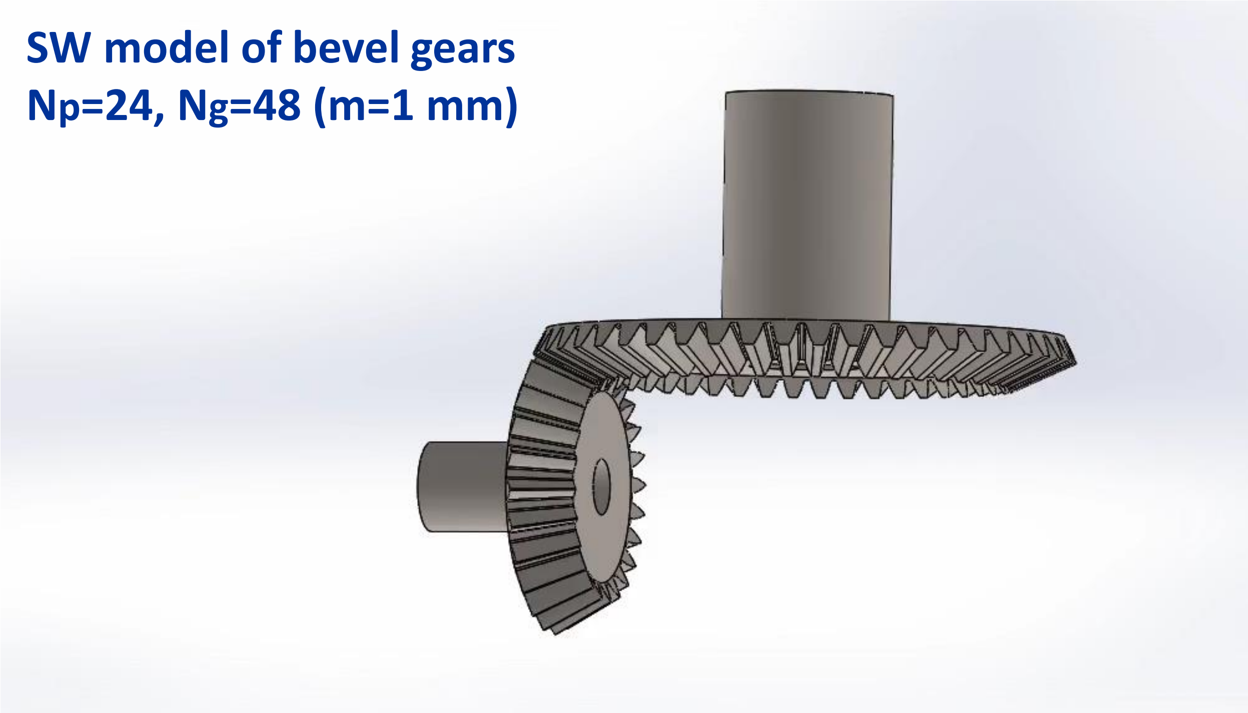


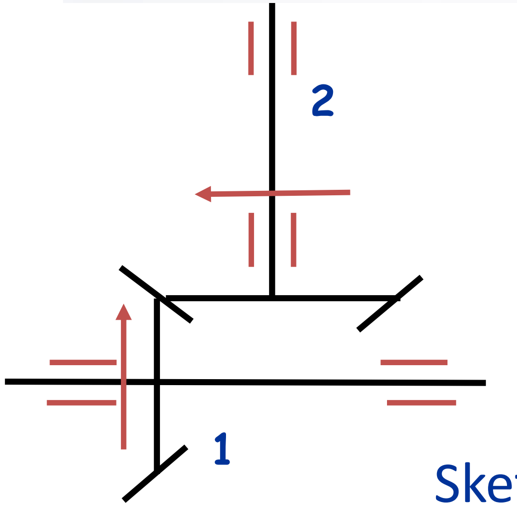
A single stage gearbox

A two stage gearbox

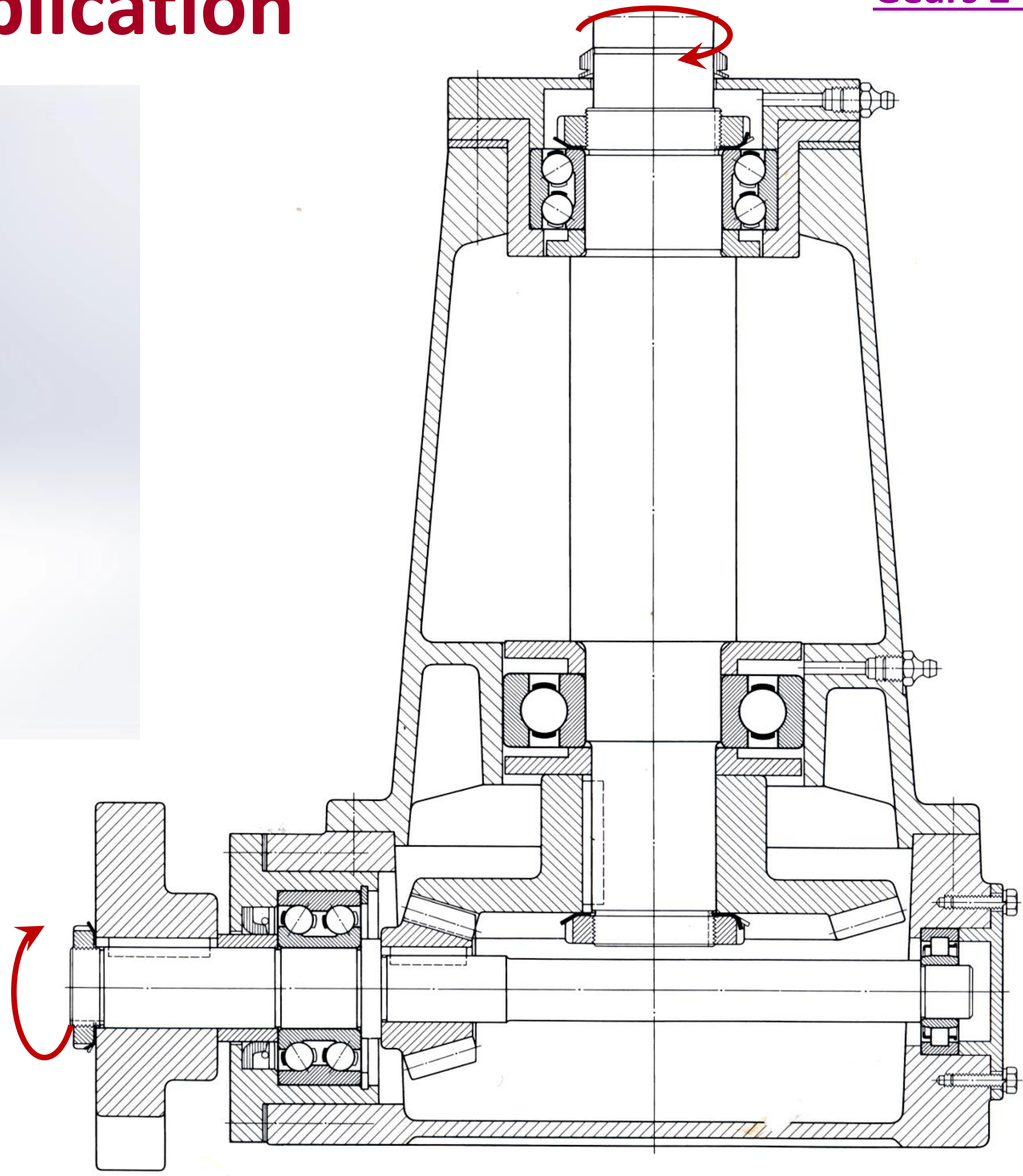
Gear train application

SW model of bevel gears
Np=24, Ng=48 (m=1 mm)





Sketch of bevel gear unit

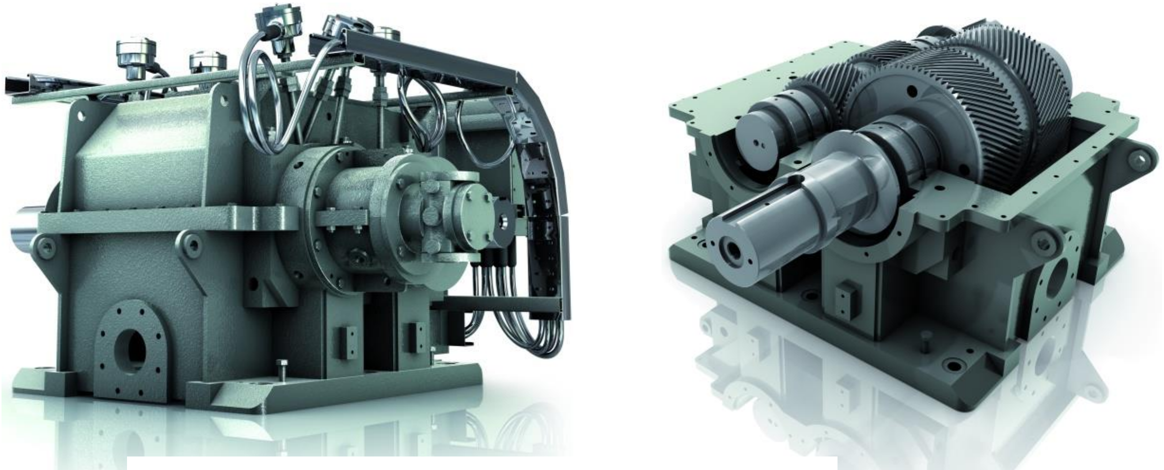


SKF **bevel gear unit**

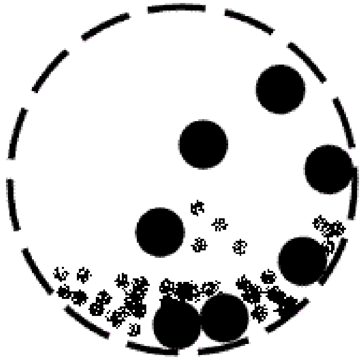
Gearbox applications: gearboxes

by David Brown Santasalo https://dbsantasalo.com/

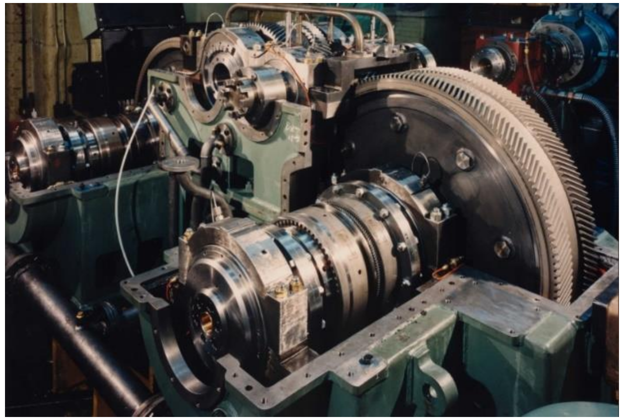
Ball Mill drive gearboxes for
>250 tons & **>14 m** in diameter



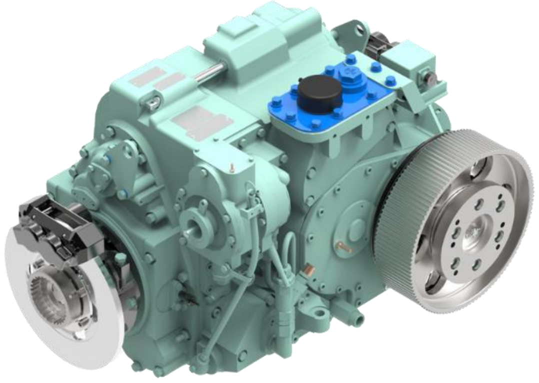
High speed gearboxes up to
54MW & **50,000 rpm**



Operation of ball mills
https://en.wikipedia.org/wiki/Ball_mill



Marine gear transmission



TN15E+ transmission for tracked vehicles

Simple & compound train summary

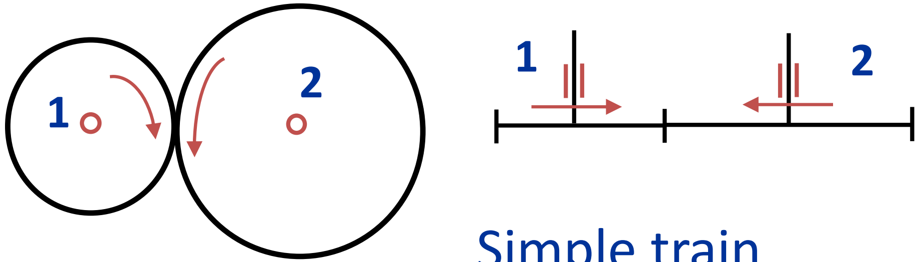
Sketch drawings & basic equations

$$Z = \frac{\omega_{In}}{\omega_{Out}} = \pm \frac{product\ of\ number\ of\ teeth\ on\ wheels}{product\ of\ number\ of\ teeth\ on\ pinions}$$

Simple train: (each shaft carries one gear)

$$Z = \frac{\omega_1}{\omega_2} = -\frac{D_2}{D_1} = -\frac{N_2}{N_1}$$

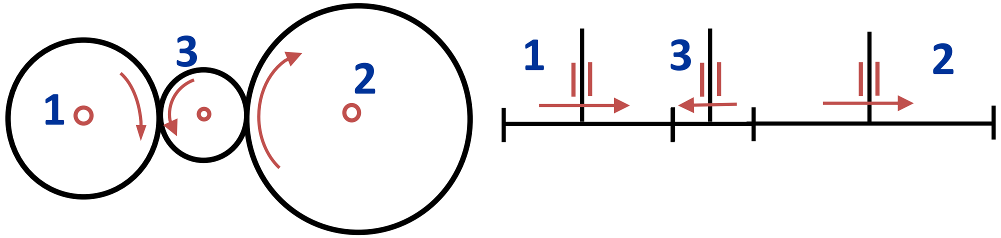
$$C = \frac{1}{2}\left(D_1 + D_2\right) = \frac{1}{2}m\left(N_1 + N_2\right)$$



Simple train

$$Z = \frac{\omega_1}{\omega_2} = \frac{D_2}{D_1} = \frac{N_2}{N_1}$$

$$C = \frac{1}{2}\left(D_1 + D_2 + 2D_3\right) = \frac{1}{2}m\left(N_1 + N_2 + 2N_3\right)$$



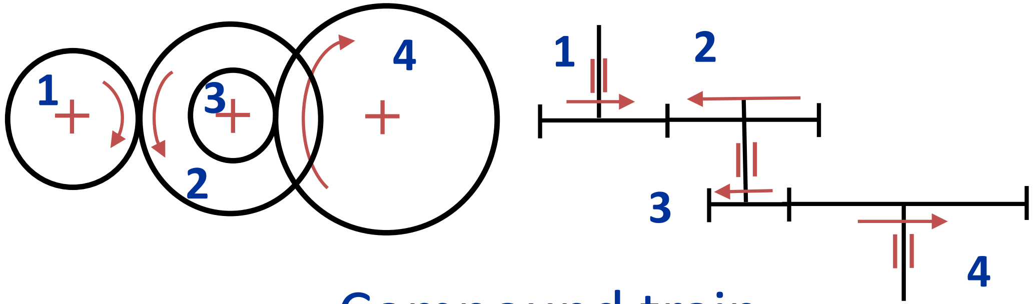
Simple train with an idler gear

Compound train: (at least one shaft carries two gears)

$$Z = \frac{\omega_1}{\omega_4} = \frac{N_2 N_4}{N_1 N_3}$$

$$C_1 = \frac{1}{2}\left(D_1 + D_2\right) = \frac{1}{2}m\left(N_1 + N_2\right)$$

$$C_2 = \frac{1}{2}\left(D_3 + D_4\right) = \frac{1}{2}m\left(N_3 + N_4\right)$$



Compound train

Worked example 1

- Draw a top view of sketch of the gear trains shown in Figs 1 and 2;
- Determine the rotating speed and direction of the **output shaft E** of the gear trains shown in Figs 1 and 2. The rotating speed of the **input shaft A** is 2000rpm. The numbers of teeth are N_A=20, N_B=40, N_C=15, N_D=25 and N_E=30, respectively.

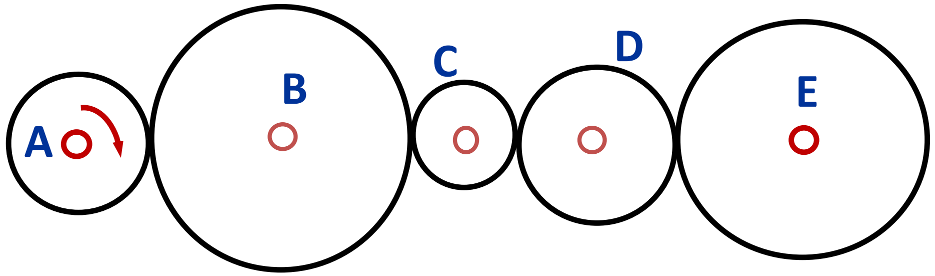


Fig 1

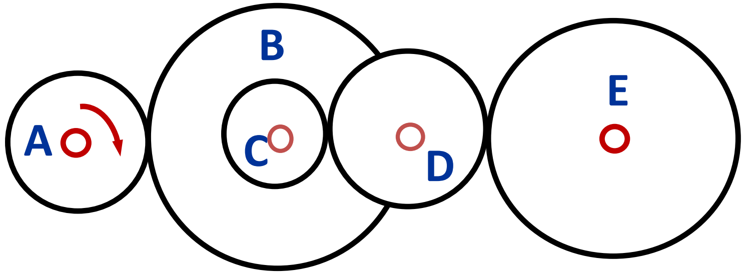


Fig 2

Worked example 1: Solution

- Draw a top view of schematic of the gear trains shown in Figs 1 and 2;
- Determine the rotating speed and direction of the **output shaft E** of the gear trains shown in Figs 1 and 2. The rotating speed of the **input shaft A** is 2000 rpm. The numbers of teeth are $N_A=20$, $N_B=40$, $N_C=15$, $N_D=25$ and $N_E=30$, respectively.

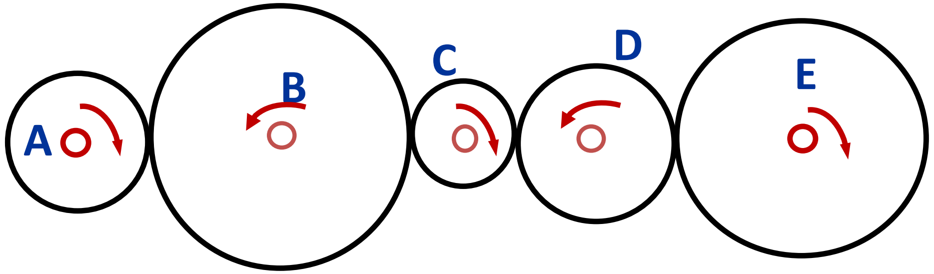


Fig 1 **Clockwise**

$$Z = \frac{\omega_A}{\omega_E} = \frac{N_E}{N_A} = \frac{30}{20} = 1.5$$

Therefore,
$$\omega_E = \frac{\omega_A}{Z} = \frac{2000}{1.5} = 1333.3(rpm)$$

$$Z = \frac{\omega_A}{\omega_E} = -\frac{N_B \cdot N_E}{N_A \cdot N_C} = -\frac{40\times30}{20\times15} = -4$$

Therefore,
$$\omega_E = \frac{\omega_A}{Z} = \frac{2000}{-4} = -500(rpm)$$

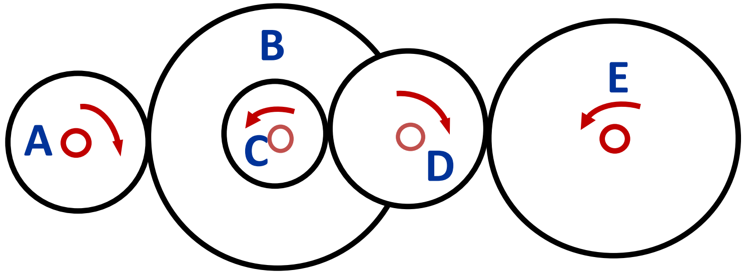


Fig 2 **Anti-Clockwise**

Reverted train

- **Reverted train** is a **compound train**, in which both the input and output shafts have the same line of axes. So the central distance must be the same for both pairs of gears.

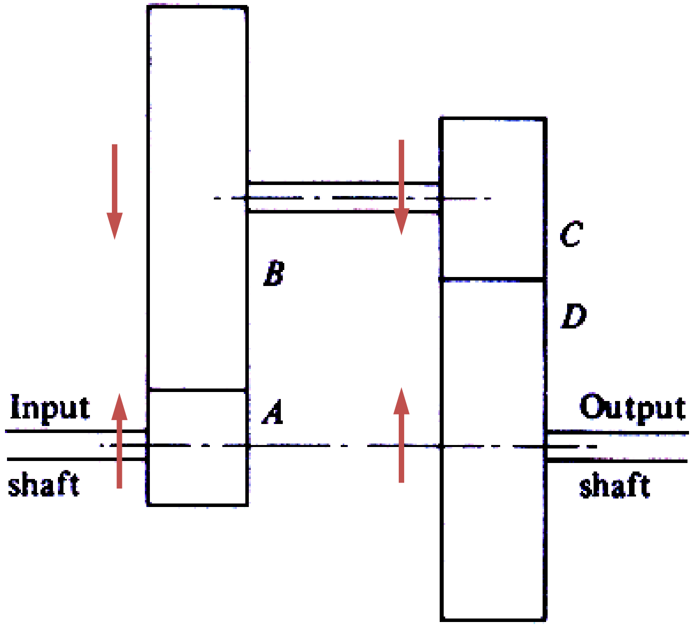
- **Velocity ratio:**

$$Z = \frac{\omega_A}{\omega_D} = \frac{N_B \cdot N_D}{N_A \cdot N_C}$$

$$2C = m_1(N_A + N_B) = m_2(N_C + N_D)$$

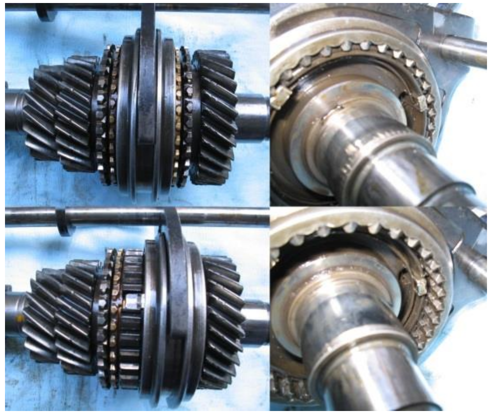
If $m_1 = m_2$, then

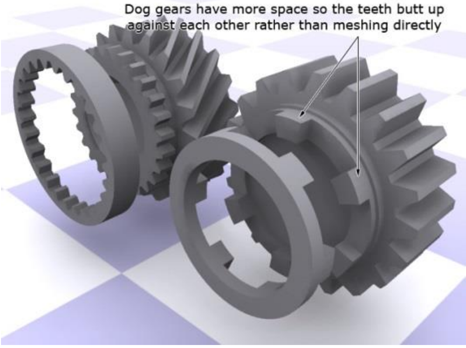
$$(N_A + N_B) = (N_C + N_D)$$



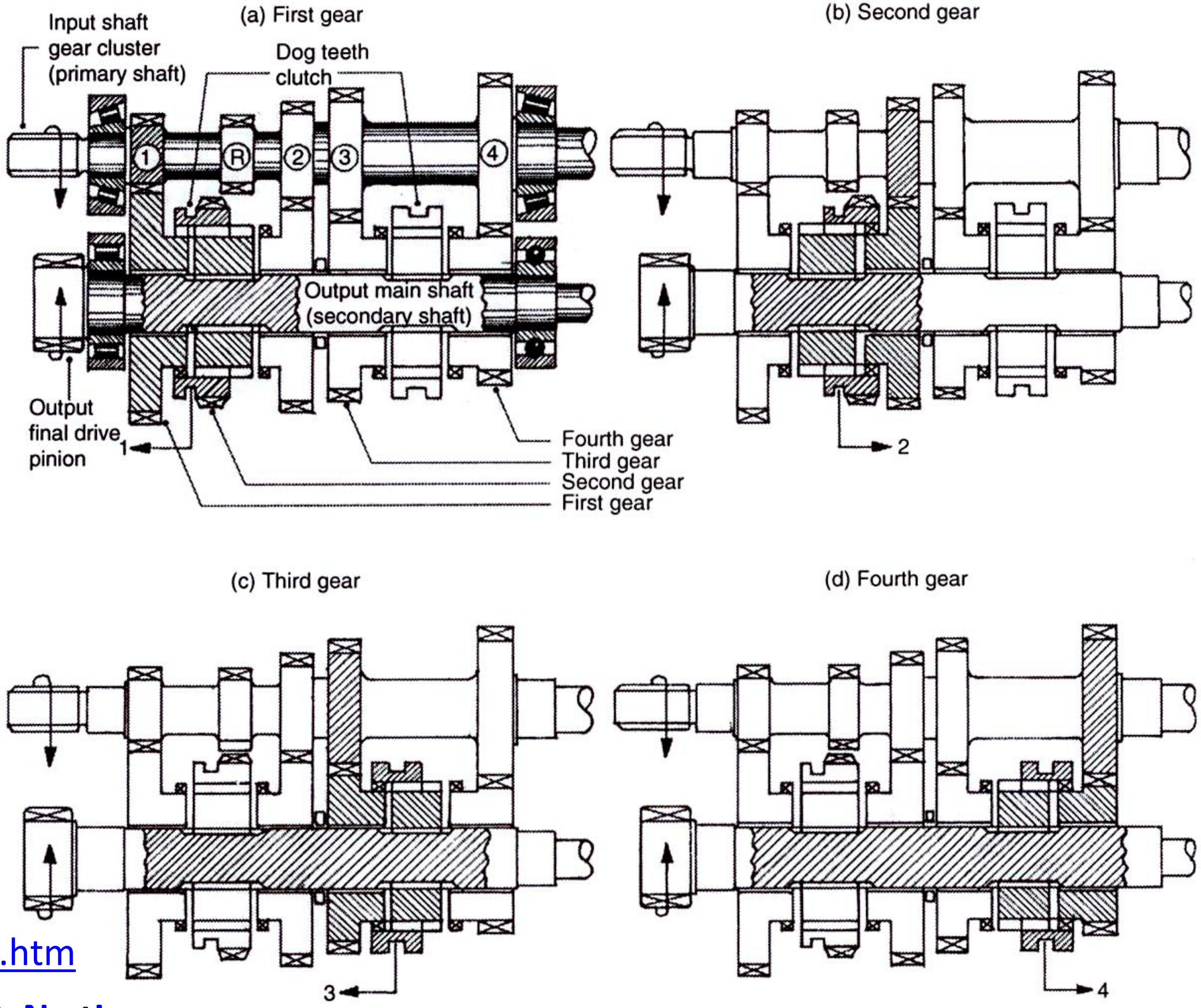
A reverted train

Application case 1: Synchromesh manual gearbox





Dog teeth clutch

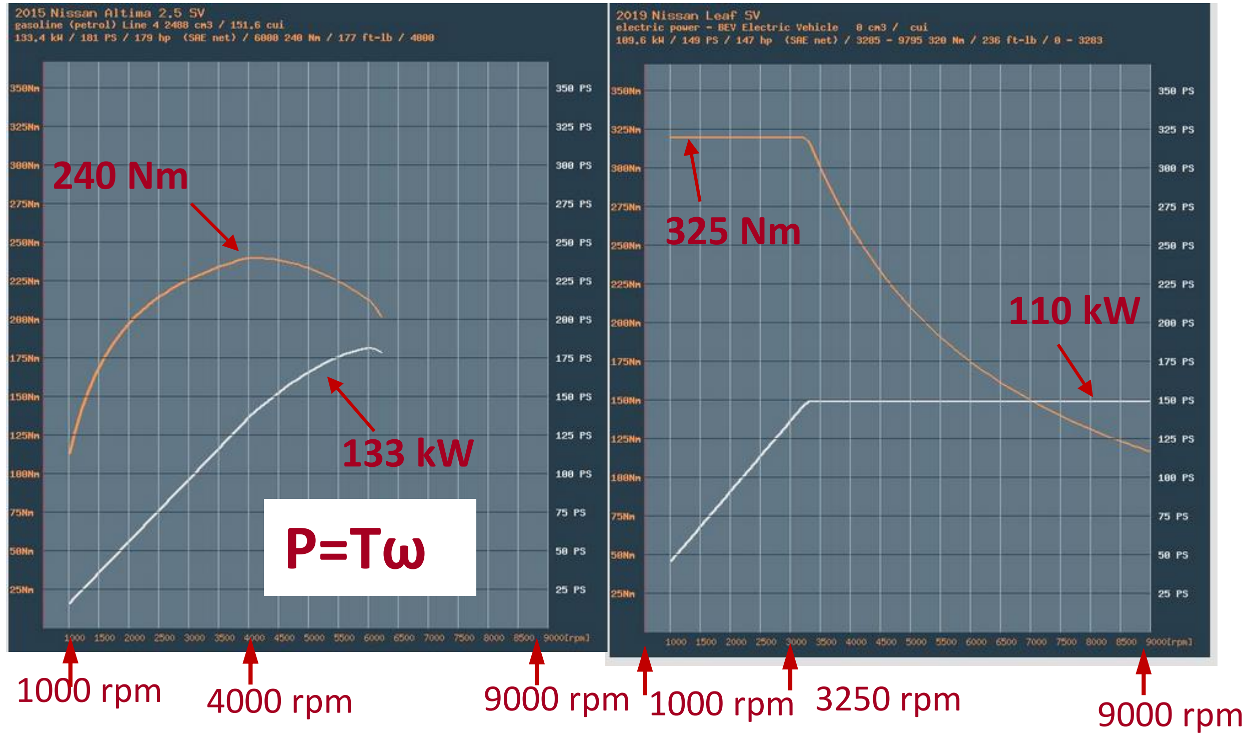


http://www.howstuffworks.com/transmission4.htm

https://www.youtube.com/watch?v=wCu9W9xNwtI

Synchromesh gearbox by Childs, pp119

Application case 2: A gearbox for electric cars (Nissan Leaf)

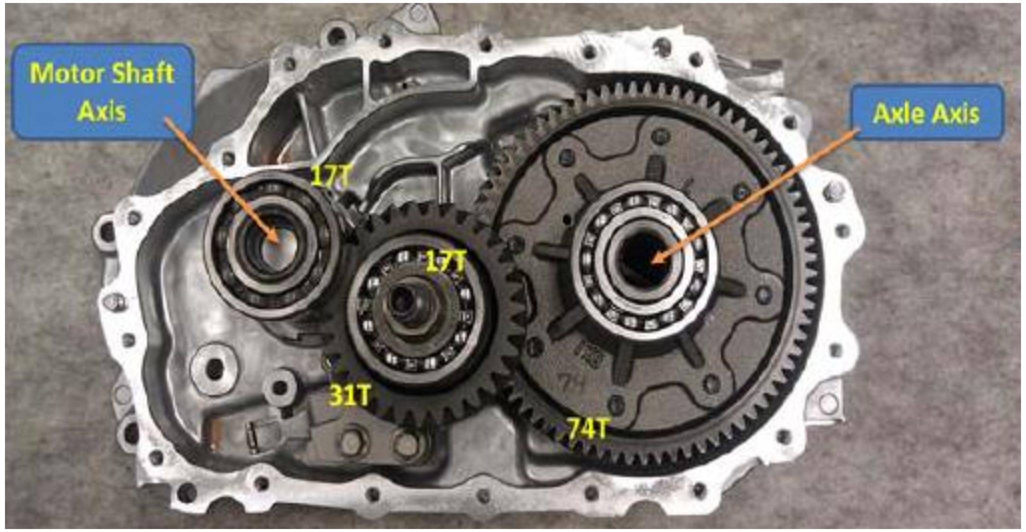


2015 Nissan Altima 2.5L SV (petrol) Power & Torque curves

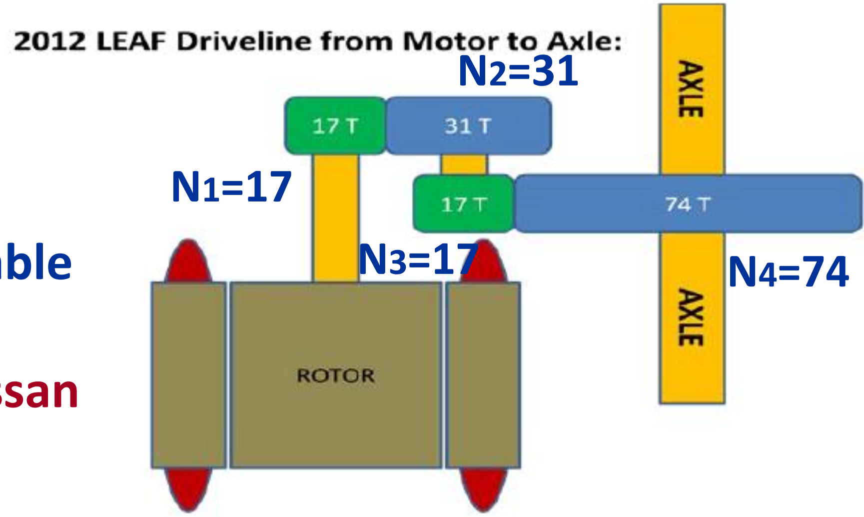
2019 Nissan Leaf Power & Torque curves

https://www.automobile-catalog.com/curve/2015/2295110/nissan_altima_2_5_sv.html

https://www.automobile-catalog.com/curve/2019/2617175/nissan_leaf_sv.html



Should you be able to calculate the gear ratio of Nissan Leaf gearbox?



“Traction drive & gearing design comparisons for multiple manufacturers & models”, An Oak Ridge National Lab report, USA, 2013

Gears 2

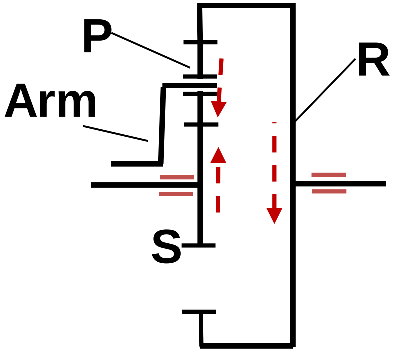
End of Part 1

Gears 2

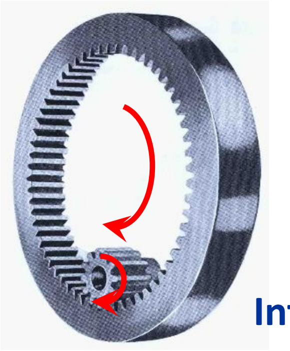
Part 2

Planetary or epicyclic trains

- A typical **planetary (epicyclic) train** includes a sun gear (**S**), an **arm** or planet carrier, one or more planet gears (**P**) often meshing with an internal ring gear (**R**).

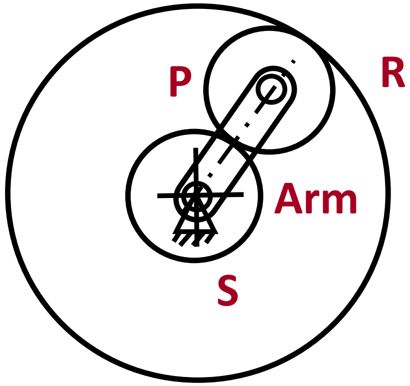


http://kmoddl.library.cornell.edu/model.php?m=43



Internal gear

Planetary gear system for geared turbo fan engine



Planetary train

Planetary train

- A **planetary train** is an unconventional system as it has ***two degrees of freedom*** ***(number of independent motions of a system)***.

- Velocity ratio:

$$Z = \frac{\omega_F - \omega_A}{\omega_L - \omega_A} = \pm \frac{product\ of\ \ number\ of\ \ teeth\ on\ wheels}{product\ of\ \ number\ of\ \ teeth\ on\ \ pinions}$$

where ω_F – speed of the First (input) gear,

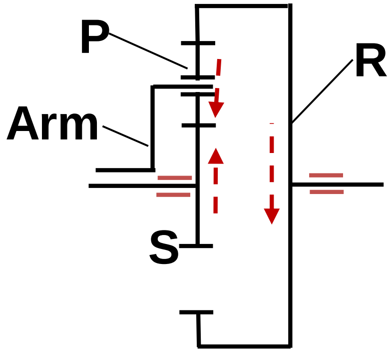
ω_L – speed of the Last (output) gear,

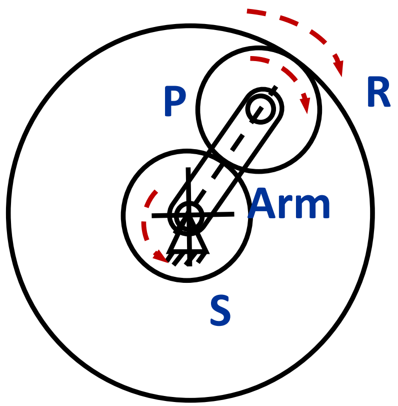
ω_A – speed of the arm or planet carrier.

In the case shown in figure,

$$Z = \frac{\omega_S - \omega_A}{\omega_R - \omega_A} = -\frac{N_P N_R}{N_S N_P} = -\frac{N_R}{N_S}$$

Planetary train is a key element of automatic transmission in cars and widely used in many other applications, e.g. wind turbine.

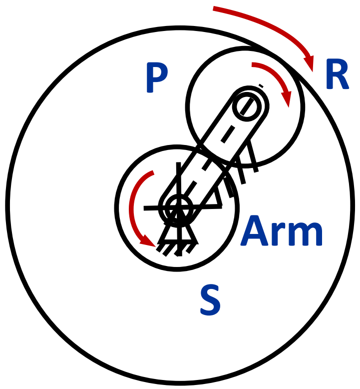


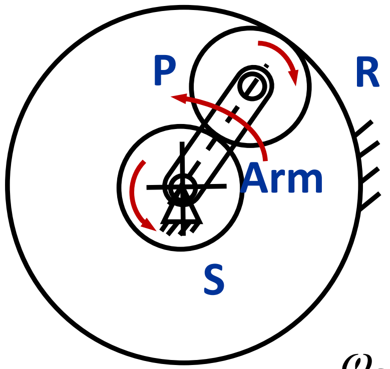


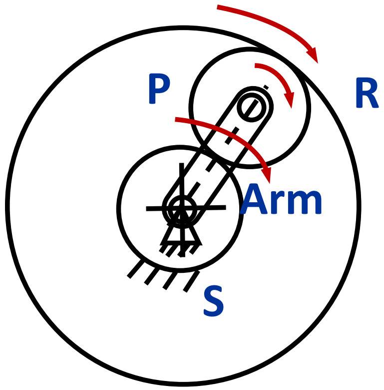
A planetary train

Planetary train

- Three arrangements of a **planetary train**,
  - **Arm** is fixed (simple train): an input of the sun gear gives the output of the ring gear in the opposite direction at reduced speed.
  - **Ring gear** is fixed: an input of the sun gear gives the output of the arm in the same direction at reduced speed.
  - **Sun gear is fixed**: an input of the arm gives the output of the ring gear in the same direction at increased speed.







$$Z = \frac{\omega_S - \omega_A}{\omega_R - \omega_A} = -\frac{N_R}{N_S}$$

$$Z = \frac{\omega_S}{\omega_R} = -\frac{N_R}{N_S}$$

Fixed arm

$$Z = \frac{\omega_S}{\omega_A} = 1 + \frac{N_R}{N_S}$$

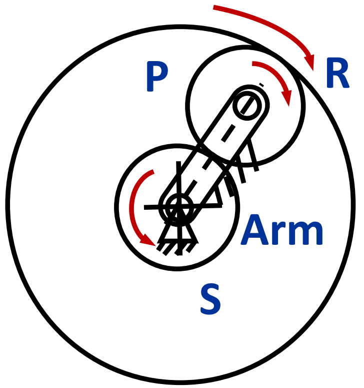
Fixed ring

$$Z = \frac{\omega_A}{\omega_R} = \frac{N_R}{N_R + N_S}$$

Fixed sun

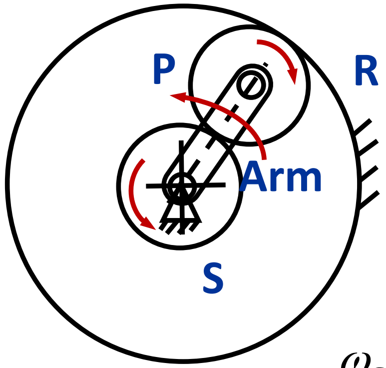
https://www.youtube.com/watch?v=pnE3Z5Zn-Zc
http://www.mekanizmalar.com/transmission.shtml

Planetary train



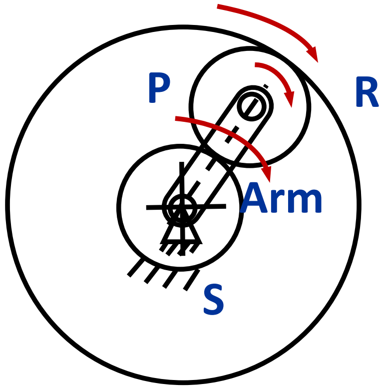
$$Z = \frac{\omega_S}{\omega_R} = -\frac{N_R}{N_S}$$

Fixed arm



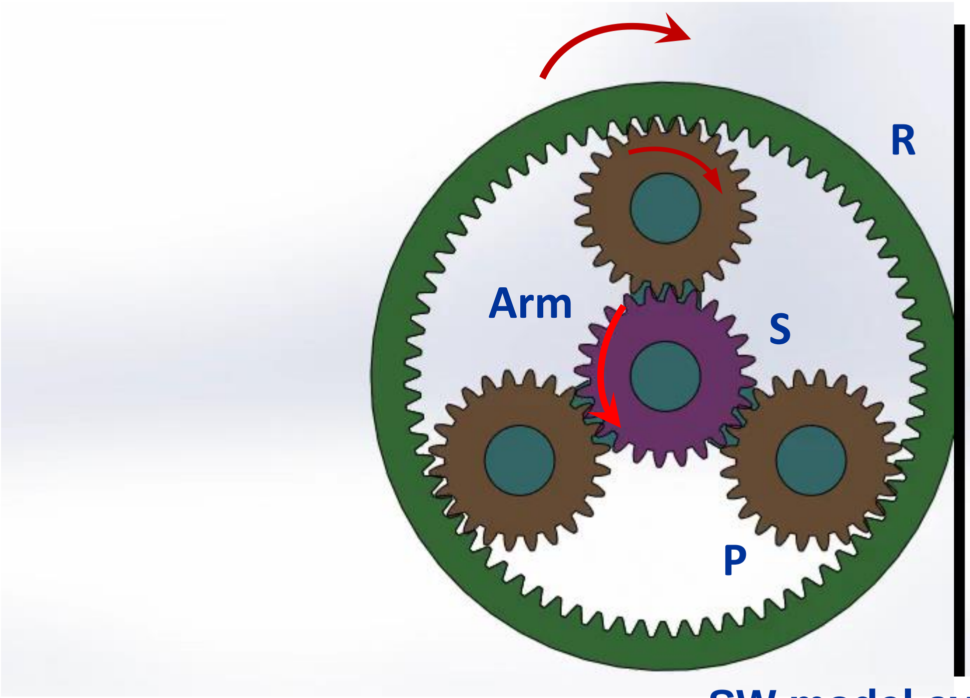
$$Z = \frac{\omega_S}{\omega_A} = 1 + \frac{N_R}{N_S}$$

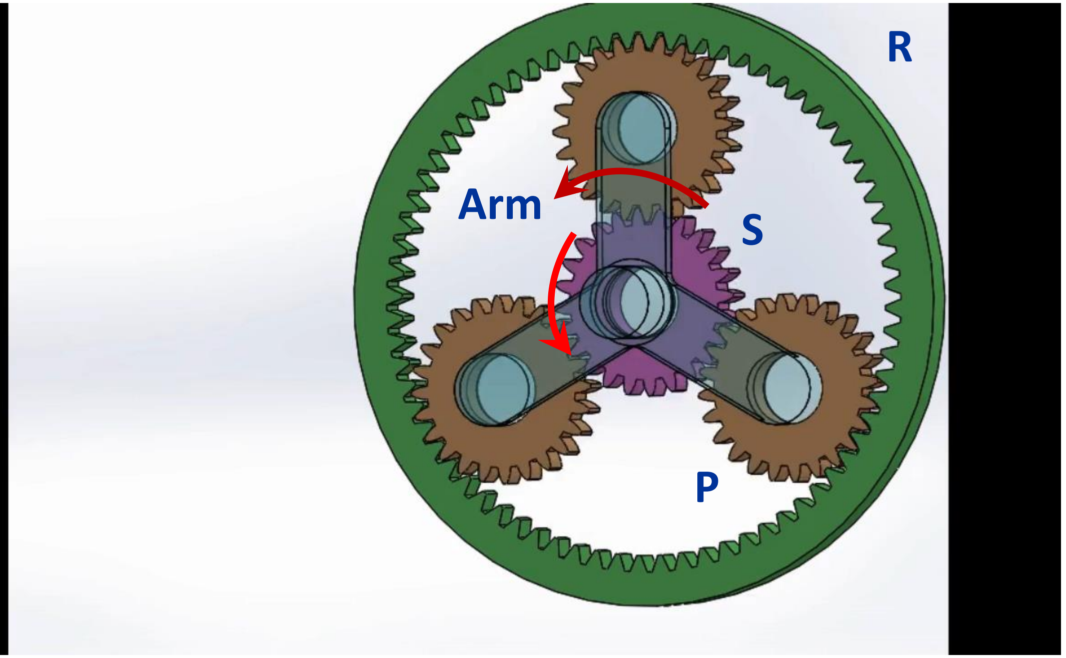
Fixed ring



$$Z = \frac{\omega_A}{\omega_R} = \frac{N_R}{N_R + N_S}$$

Fixed sun



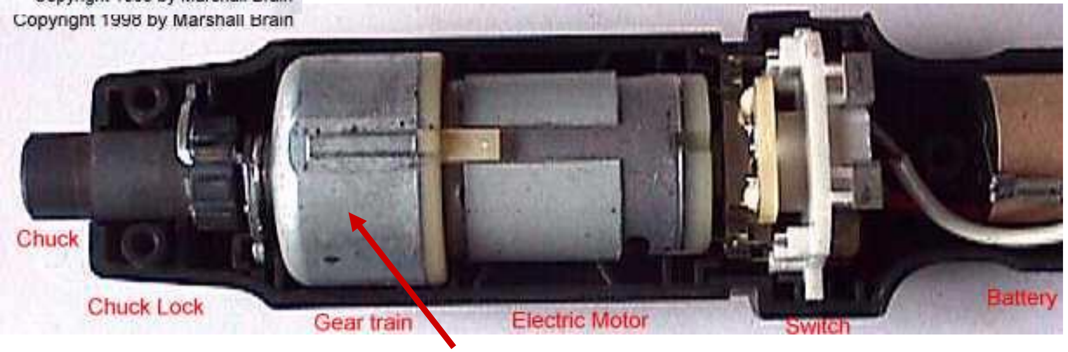


SW model available on Moodle

A worked example of planetary train: A hand drill

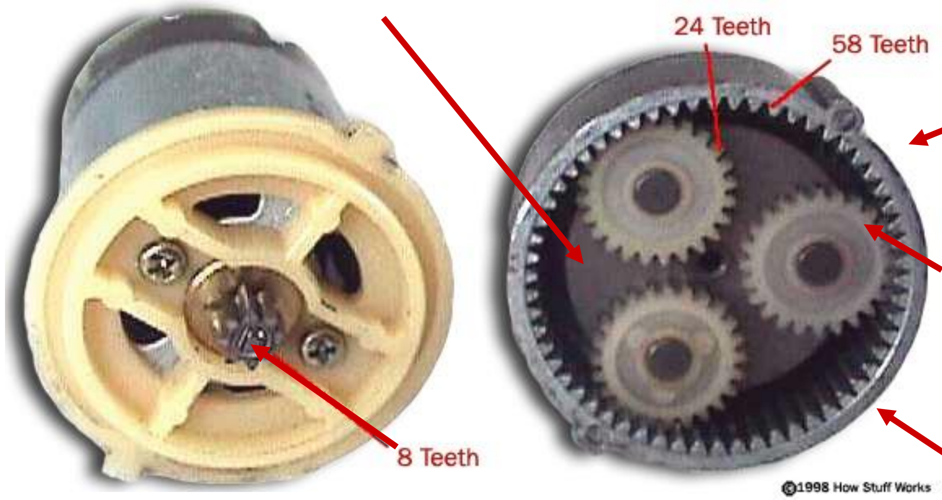
Advantages:

- compact
- Input and Output Shafts in line



Carrier Plate (attached to output shaft)

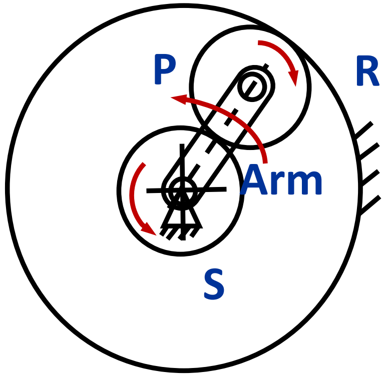
Planetary Gearset



Planets rotate along with Carrier Plate

Ring is stationary

Sun connected to motor

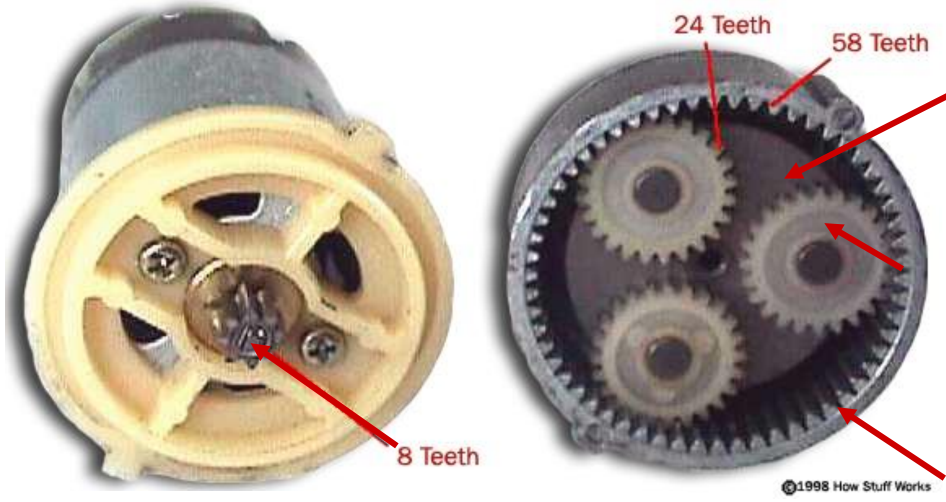


$$Z = \frac{\omega_S}{\omega_A} = 1 + \frac{N_R}{N_S}$$

Fixed ring

A large gear ratio can be achieved using a planetary gear set

Hand drill: Solution

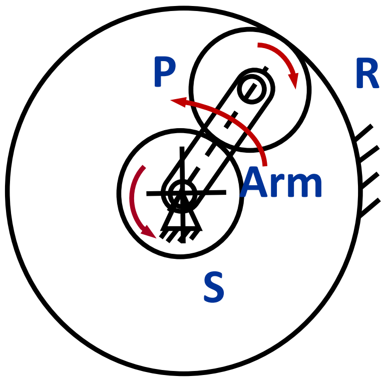


Carrier Plate (attached to output shaft)

Planets rotate along with Carrier Plate

Ring is stationary

Sun connected to motor



$$Z = \frac{\omega_S}{\omega_A} = 1 + \frac{N_R}{N_S}$$

Fixed ring

In the hand drill, the number of teeth of the sun, planet and ring gears are $N_S = 8$, $N_P = 24$ and $N_R = 58$. To achieve an output shaft speed, $\omega_A = 200$ rpm, calculate the input shaft speed, i.e. the speed of the sun gear, ω_S.

Using the above gear ratio equation,

Z=8.25

$$\omega_S = \left(1 + \frac{N_R}{N_S}\right)\omega_A = \left(1 + \frac{58}{8}\right) \times 200 = 1650\,(rpm)$$

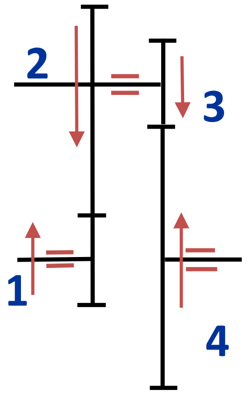
What will happen if we connect two or more planetary gear sets together?

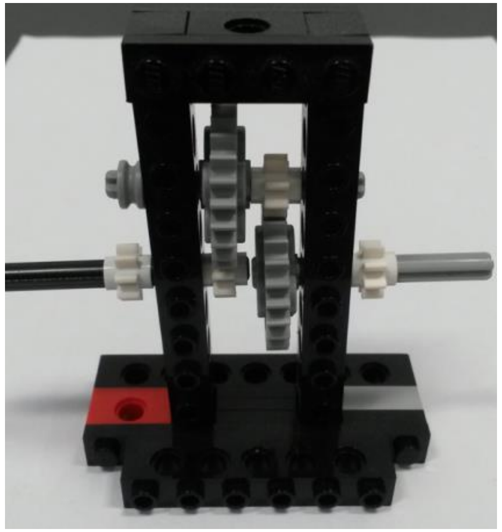
https://www.youtube.com/watch?v=wKTARAH2dIo

How to design a planetary gear train?

A Lego example

Just add in a planet carrier or arm to achieve a **two DoF system**

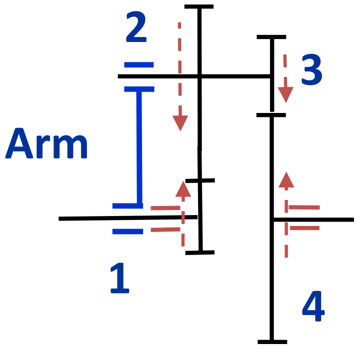




$$Z = \frac{\omega_1}{\omega_4} = \frac{N_2 N_4}{N_1 N_3}$$

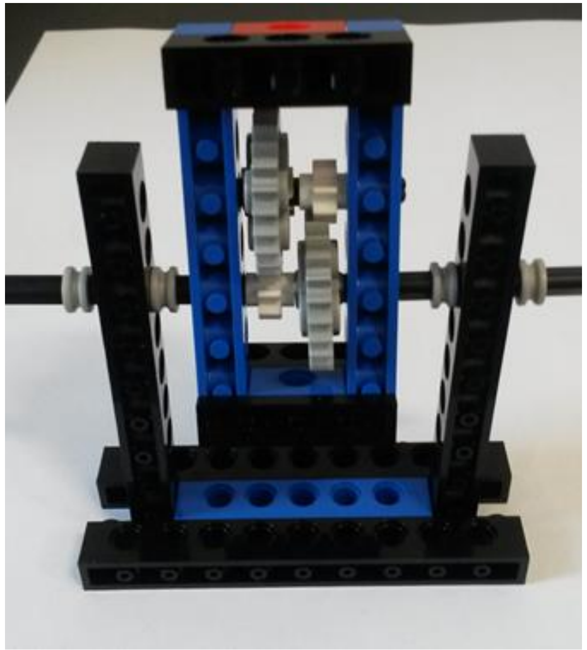
Reverted train

$N_1 = N_3 = 8$,
$N_2 = N_4 = 24$



$$Z = \frac{\omega_1 - \omega_A}{\omega_4 - \omega_A} = \frac{N_2 N_4}{N_1 N_3}$$

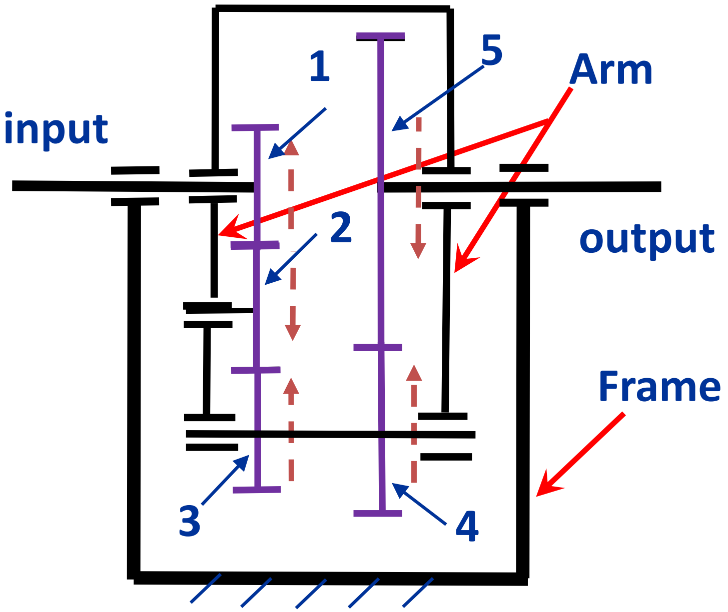
Planetary train

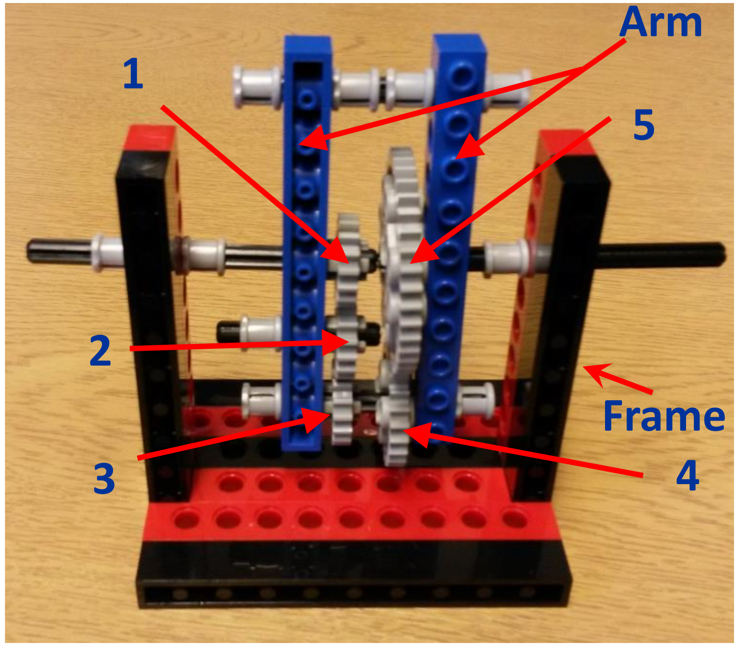


How to design a planetary gear train?

Another Lego example

- Schematic of a Lego gear train





N1=N2=N3=18, N4=24, N5=48

- Gear ratio equation

$$Z = \frac{\omega_1 - \omega_A}{\omega_5 - \omega_A} = -\frac{N_3 N_5}{N_1 N_4}$$

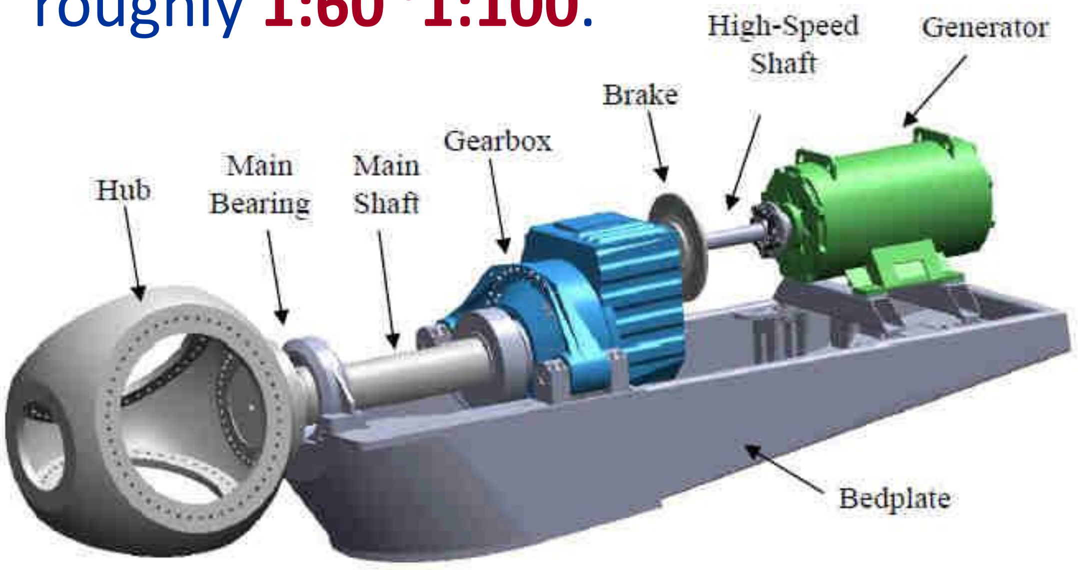
To fix the arm, $Z = \frac{\omega_1 - 0}{\omega_2 - 0} = -\frac{48}{24} = -2$

To fix shaft (5), $Z = \frac{\omega_1 - \omega_A}{0 - \omega_A} = -\frac{48}{24} = -2$

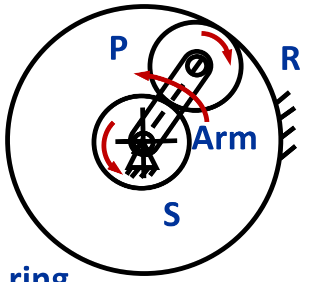
$$Z = \frac{\omega_1}{\omega_A} = 3$$

Applications of planetary train: **wind turbine gearbox**

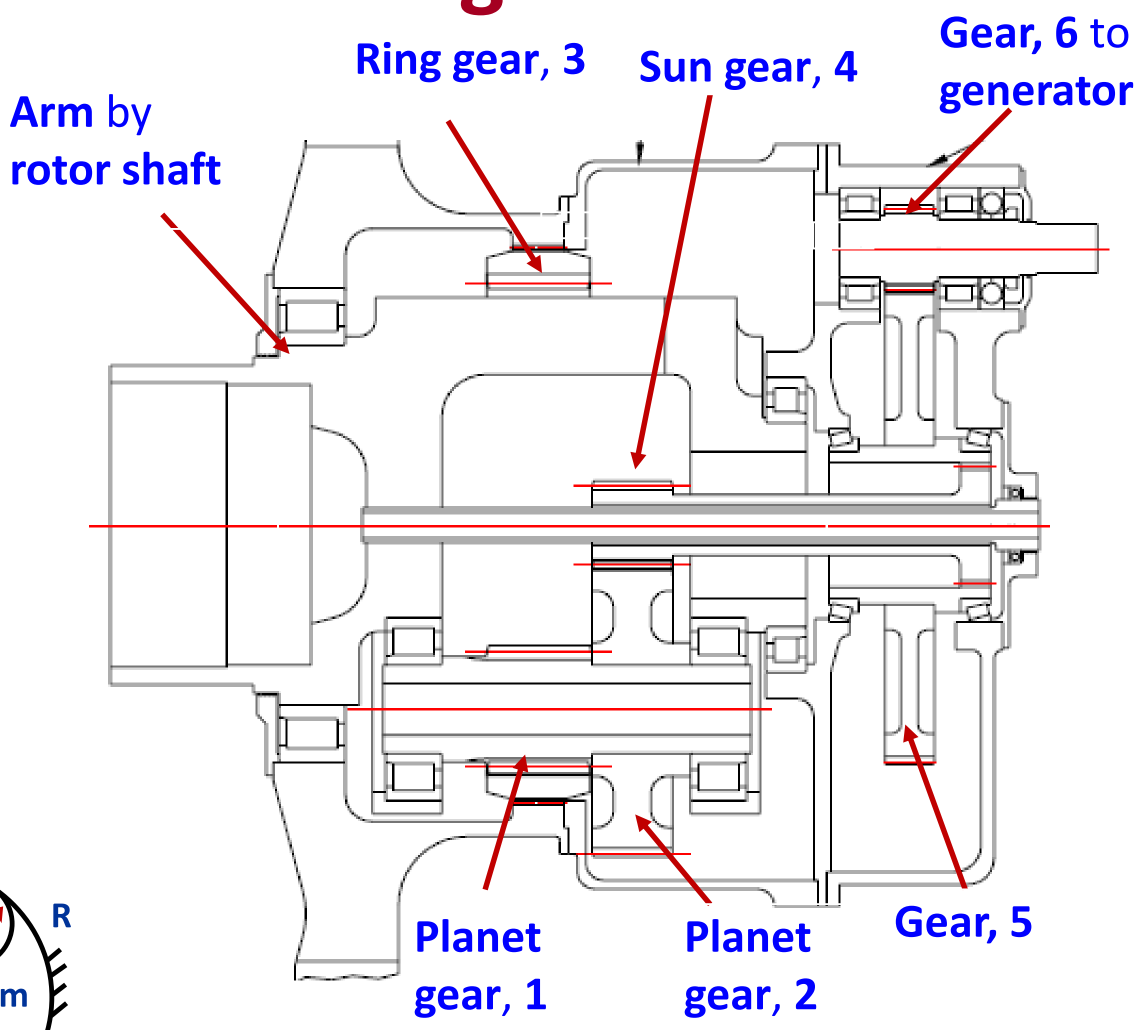
A **gearbox in wind turbine** allows **low speed** (15~25rpm) **high torque power** to be increased to **high speed** (1000~2500rpm) **low torque power** to generate electricity and this can be achieved by a **gear ratio** of roughly **1:60~1:100**.



Modular drive train configuration
http://www.nrel.gov/wind/



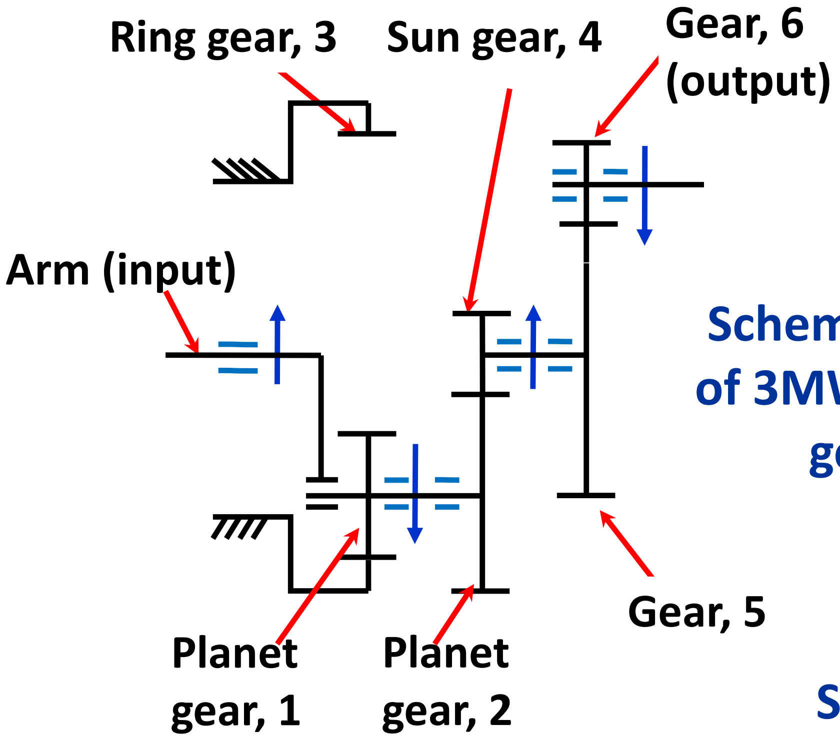
$$Z = \frac{\omega_S}{\omega_A} = 1 + \frac{N_R}{N_S}$$



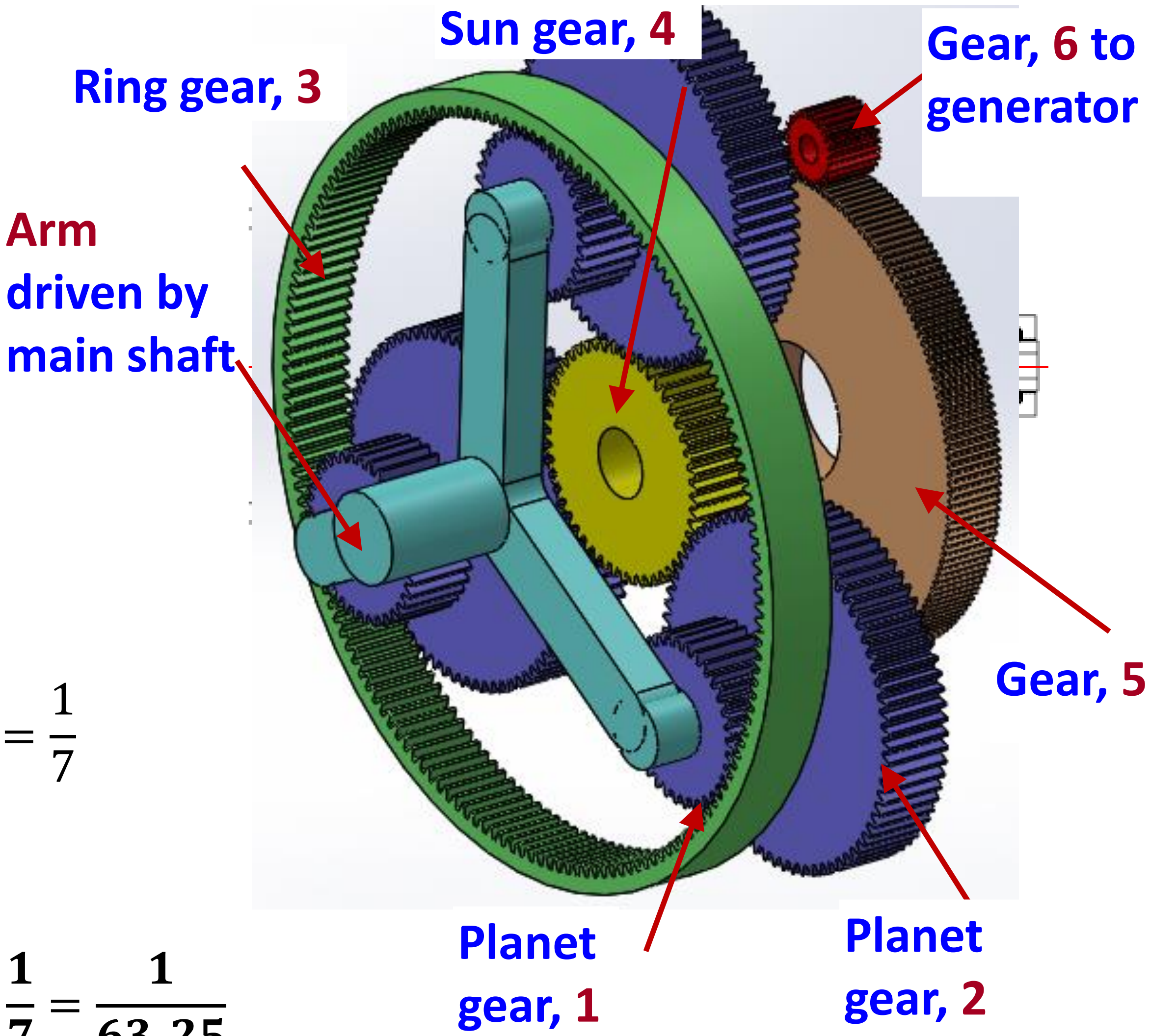
A 3MW compound planetary helical gearbox,
http://www.nrel.gov/wind/

Applications: **wind turbine gearbox**

Estimated Numbers of teeth: $N_1=35$, $N_2=75$, $N_3=150$, $N_4=40$, $N_5=175$, $N_6=25$



Schematic drawing of 3MW compound gear train



A 3MW compound planetary helical gearbox,
A SW model of WT gearbox

Planetary train:

$$\frac{\omega_4-\omega_A}{\omega_3-\omega_A}=-\frac{N_2N_3}{N_4N_1}$$

As Ring gear 3 fixed

ω3 = 0,

$$Z_1=\frac{\omega_A}{\omega_4}=\frac{1}{1+\frac{N_2N_3}{N_4N_1}}=\frac{1}{9.036}$$

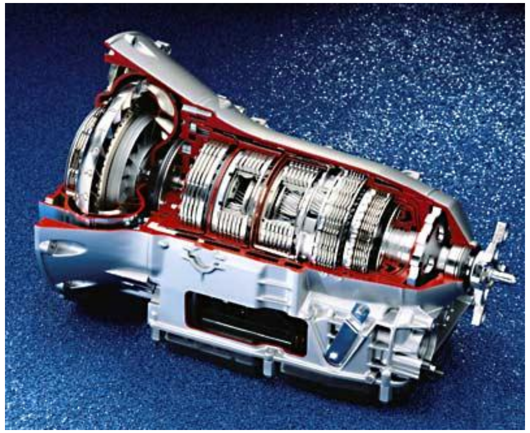
Simple train:

$$Z_2=\frac{\omega_5}{\omega_6}=\frac{N_6}{N_5}=\frac{25}{175}=\frac{1}{7}$$

Overall ratio:

$$Z=Z_1Z_2=\frac{1}{9.036}\times\frac{1}{7}=\frac{1}{63.25}$$

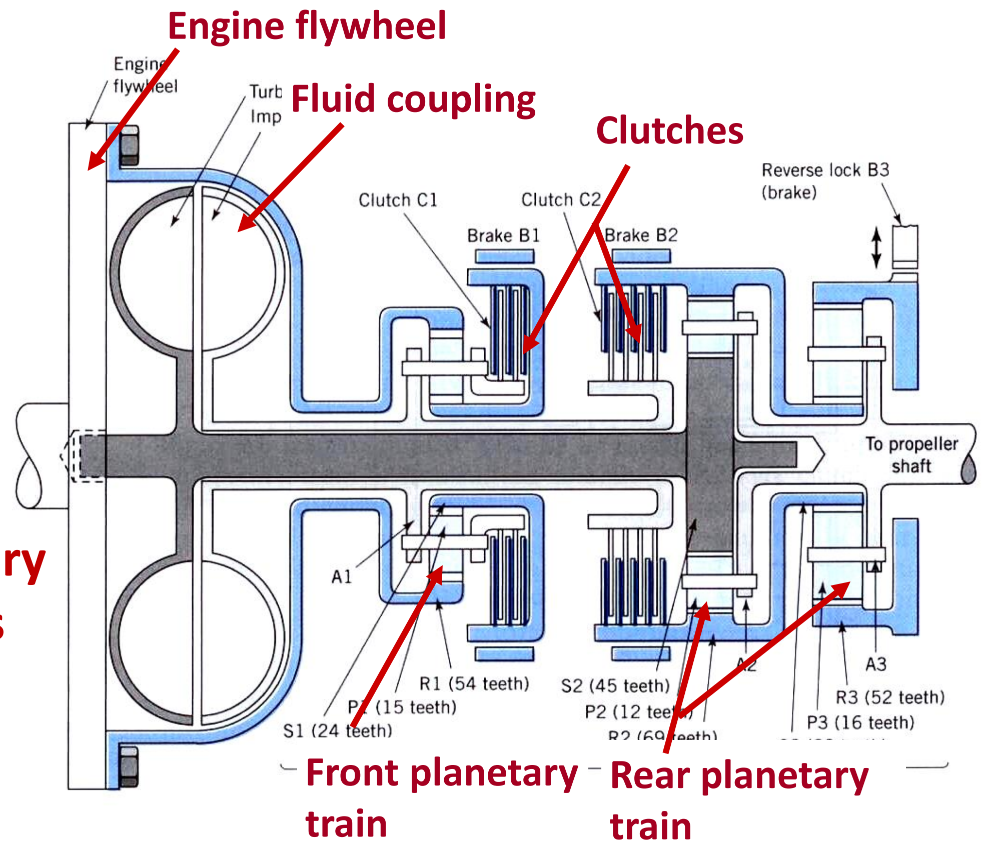
Other applications: **Automatic gearbox in cars & helicopters**



Mercedes CLK, automatic transmission

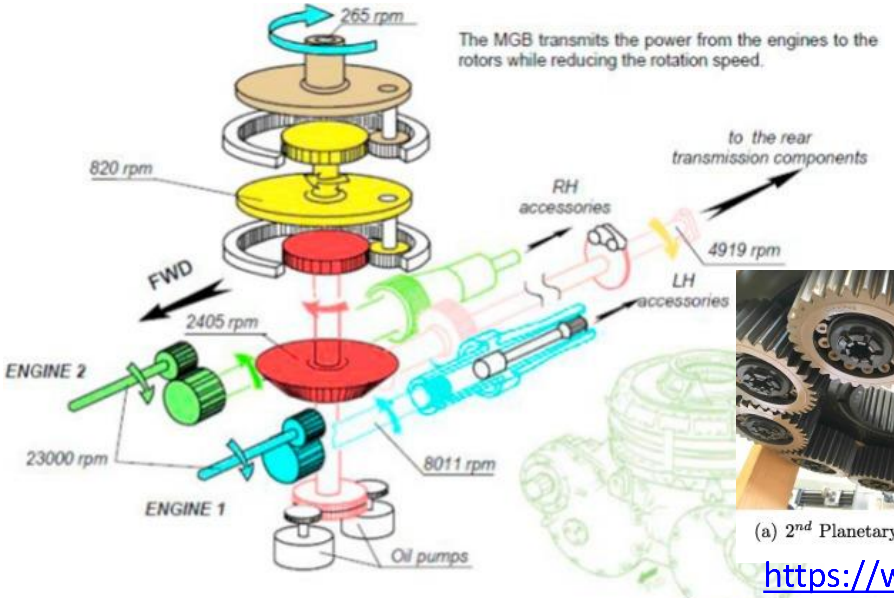
GE Hydra-Matic 4T45 FWD

Fluid coupling drives 3 planetary trains plus brakes and clutches



Schematic of Hydra-Matic Transmission

https://www.youtube.com/watch?v=auQgOtveQi0



(a) 2^{nd} Planetary gears/ bearings

(b) 2^{nd} Epicyclic module ring and sun gears

https://www.sciencedirect.com/science/article/pii/S0003682X17306552#f0010

Helicopter Main Gearbox (MGB)

You don't need to know the details of this slide!

Gears 2

End of Part 2

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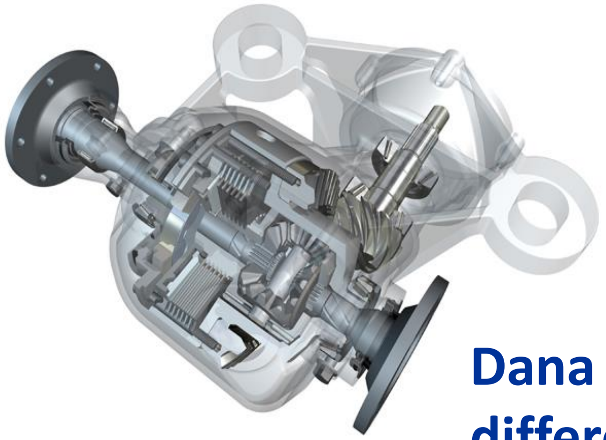
Gears 2

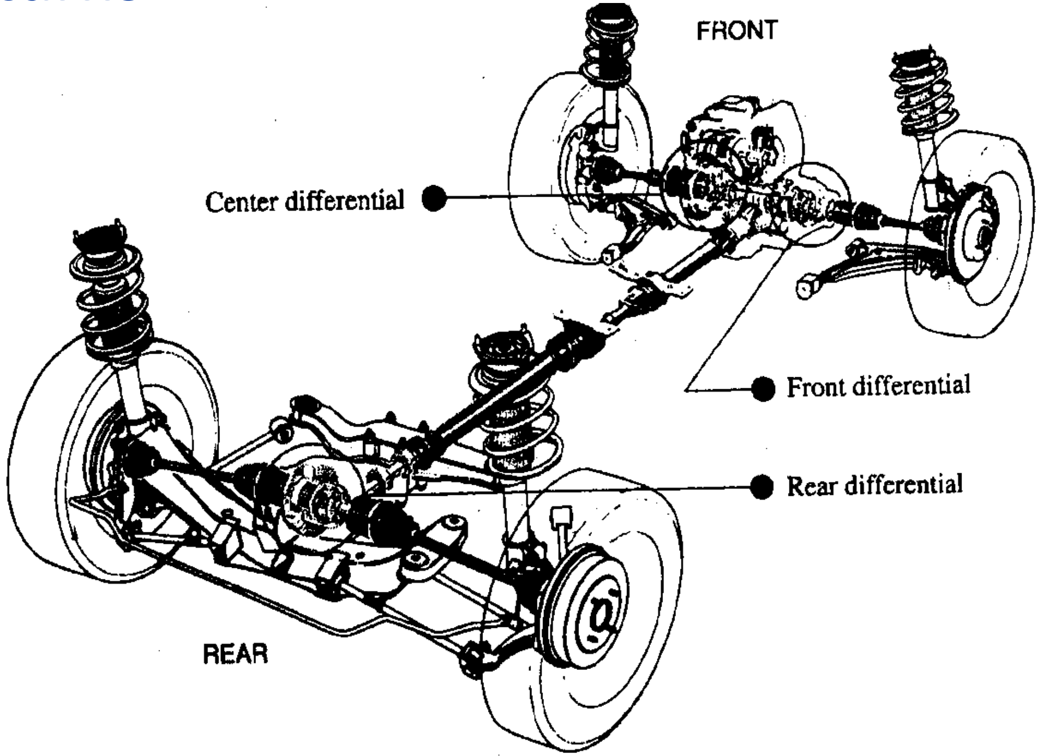
Part 3

Application of planetary train: **Differential unit**

Differential unit is an essential auto unit to split engine/electric motor power to drive wheels and to accommodate difference in speed when a car turns



Dana limited slip differential unit



An all-wheel-drive (AWD) chassis and drive train

Automotive differential unit

Application of planetary train: **Differential unit**

Differential unit (a planetary train with bevel gears):

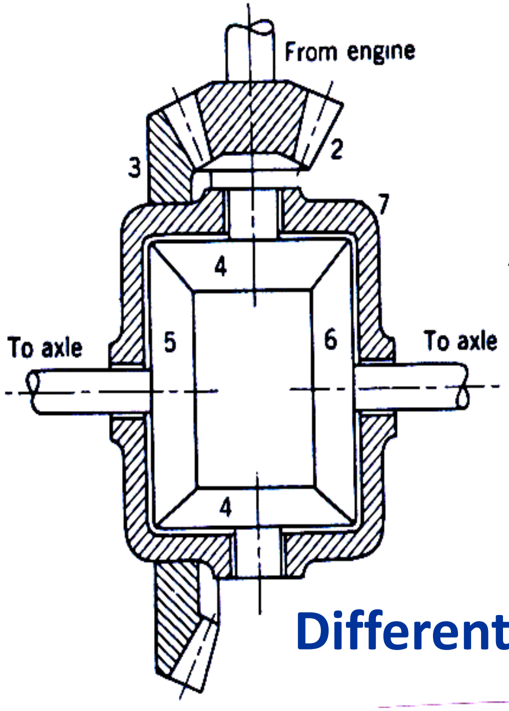
- Differential unit is used **to split engine power to the drive wheels** and **to allow different velocities of the drive wheels when turning**.

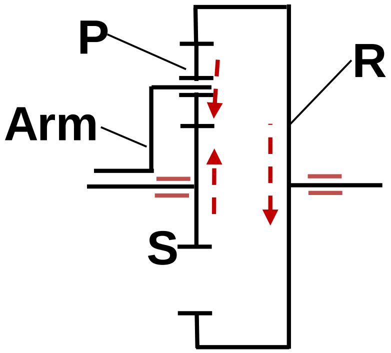
- Engine shaft drives **Gear 2** meshing with **Gear 3 (arm)**. **Gear 4 (planetary gears)** mounted on a **spider arm 7 & 3**. **Gears 5** and **6** are either the **sun** or **ring gear**.

$$\frac{\omega_5 - \omega_3}{\omega_6 - \omega_3} = -\frac{N_4}{N_5} \times \frac{N_6}{N_4} = -1$$

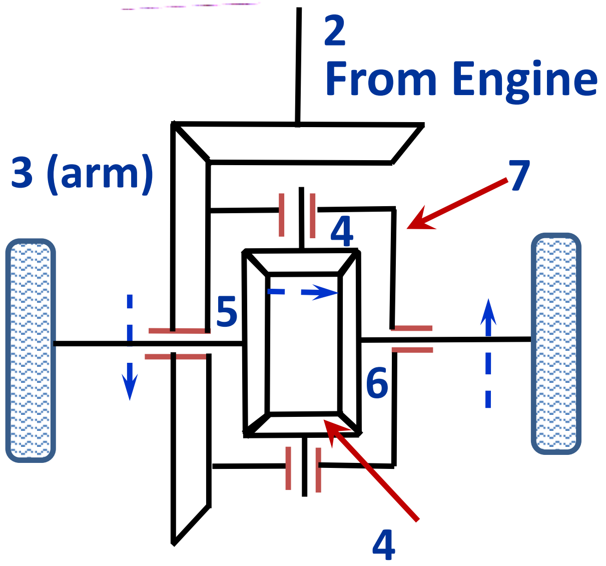
- Basic equation of differential unit is

$$\omega_5 + \omega_6 = 2 \cdot \omega_3$$



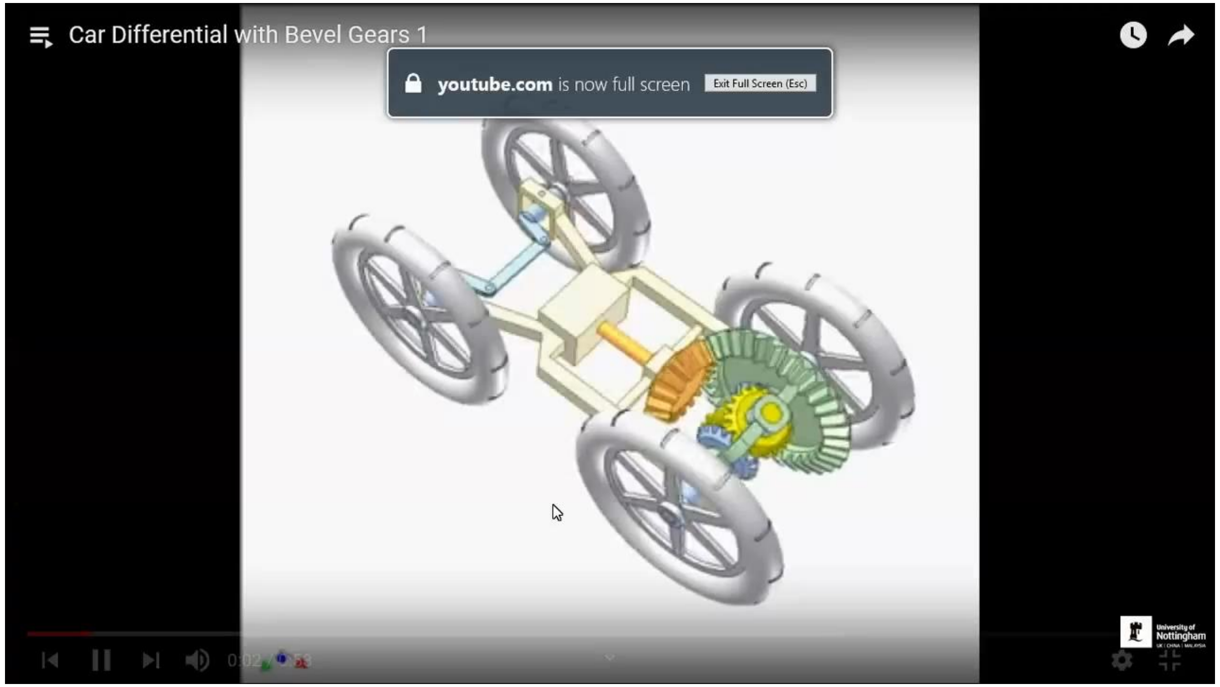


Differential unit



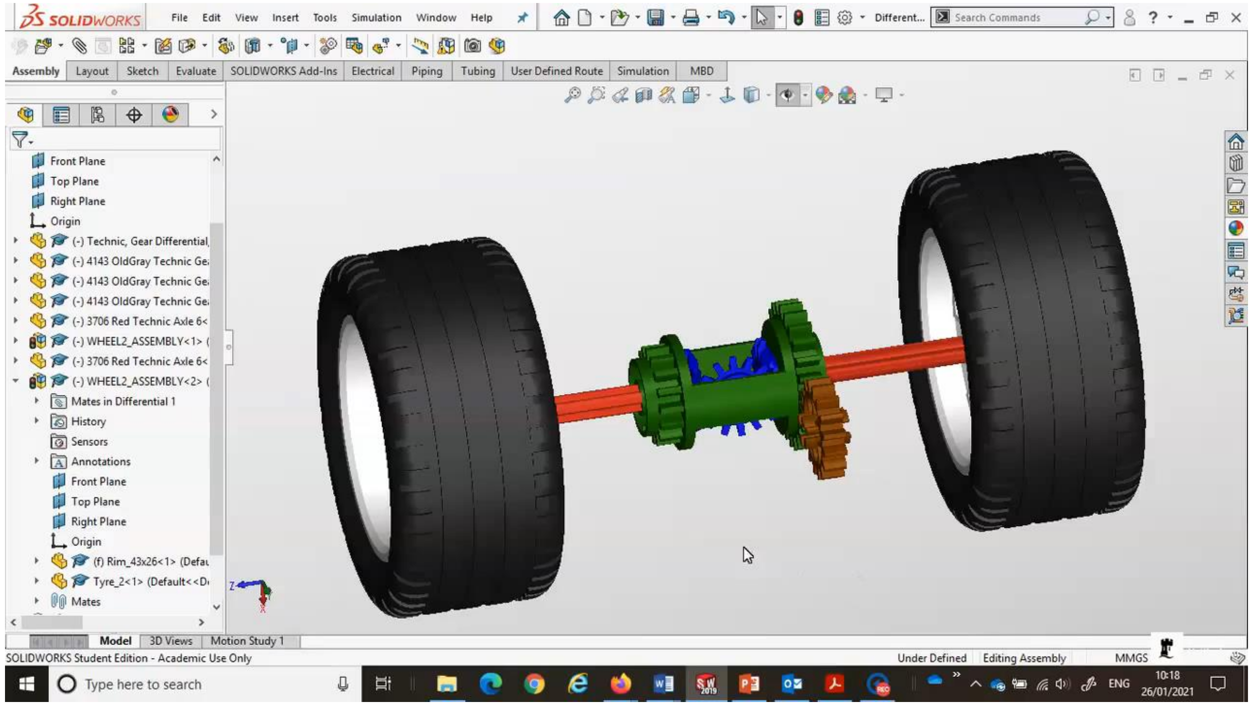
Application of planetary train: **Differential unit**

A Youtube animation of differential unit



https://www.youtube.com/watch?v=YjhzkV5Ya2k

A Lego model of differential unit



Lego parts downloaded from https://Grabcad.com/ and the SW model is available on Moodle

https://www.youtube.com/watch?v=SOgoejxzF8c

https://www.youtube.com/user/thang010146/videos

Quiz: **True or False** to each of the following statements

Gears 2

A. In a compound gear train, each shaft carries only one gear.

B. A reverted gear set is a simple gear train.

C. A planetary or epicyclic gear train has TWO degrees of freedom.

D. Differential unit is used to split engine power to the drive wheels and to allow different velocities of the drive wheels when a car turns.

E. A reduction gearbox is commonly used in a wind turbine for power generation.

F. The Nissan Leaf gearbox is a compound gear train.

Differential unit

Quiz: **True or False** to each of the following statements

Gears 2

A. In a compound gear train, each shaft carries only one gear. (false)

B. A reverted gear set is a simple gear train. (false)

C. A planetary or epicyclic gear train has TWO degrees of freedom. (true)

D. Differential unit is used to split engine power to the drive wheels and to allow different velocities of the drive wheels when a car turns. (true)

E. A reduction gearbox is commonly used in a wind turbine for power generation. (false)

F. The Nissan Leaf gearbox is a compound gear train. (true)

Summary

- To be familiar with different types of gear trains and their schematic representations

- To be able to use the basic equations to derive or calculate gear ratio of a specific gear train

$$Z = \frac{\omega_{In}}{\omega_{Out}} = \pm \frac{product\ of\ number\ of\ teeth\ on\ wheels}{product\ of\ number\ of\ teeth\ on\ pinions}$$

for all simple and compound trains

$$Z = \frac{\omega_F - \omega_A}{\omega_L - \omega_A} = \pm \frac{product\ of\ number\ of\ teeth\ on\ wheels}{product\ of\ number\ of\ teeth\ on\ pinions}$$

for all planetary trains including differential unit

- To understand the working mechanisms of a number of gear systems in application

Revision questions

- What is the difference between a simple and compound gear train?
- Can you calculate the gear ratio of a simple and compound gear train?
- Can you use the general gear ratio equation of a planetary gear train to calculate the gear ratios under three different operation conditions (e.g. with a fixed arm, ring or sun gears)?
- Based on the examples of the Lego gear kits, would you be able to design a planetary gear train?
- Do you understand the derivation and detailed calculation of the total gear ratio of the 3 MW wind turbine example?
- Differential unit is an essential auto power transmission unit in all auto vehicles, do you know its main function and how it works when a car drives on a straight road or makes a turn?

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Friday's CAE & Project Support session is to help on CAE 4 task and Individual design (Gearbox actuator) Project

In this Friday's session (4-6pm, 17th Feb), we'll also give a demo on using Solidworks to design geartrains & to do motion analysis

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Gears 2

End of session